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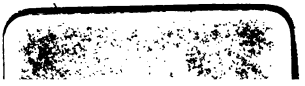
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ARITHMETIC

FOR THE USE OF SCHOOLS.

BY

EDWARD LIDDELL,

ASSISTANT TEACHER, HOMERTON COLLEGE.

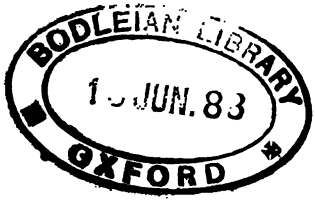
SECOND EDITION.

LONDON:

LONGMAN, GREEN, LONGMAN, AND ROBERTS.

1861.

1802. f 71



LONDON:
GEORGE UNWIN, GRESHAM STEAM PRESS,
BUCKLERSBURY, E.C.

PREFACE.

THIS Manual has been prepared at the request of several teachers. The main object has been to furnish a sufficiently large number of examples for school practice. In each set of exercises the questions have been carefully graduated according to their difficulty; and throughout the book these have been constructed, as far as possible, with direct reference to actual business.

E. L.

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ARITHMETIC.

DEFINITIONS.

1. A collection of objects of the same kind is called a ***number.***

A single object in such a collection is called a *unit.*

Numbers are of two kinds—*abstract* and *concrete.*

A *concrete number* points out the kind of unit from which it is formed. Six days is a concrete number, its unit being *one day.*

An *abstract number* does not point out the kind of its unit. Six is an abstract number, its unit being simply *one.*

Arithmetic is the science which treats of the operations by which numbers are affected.

NOTATION AND NUMERATION.

2. *Notation* is a method of writing any number in figures. In *Numeration,* numbers given in figures are required to be expressed in words.

The figures used in arithmetic, with their names and values in units, are as follows :—

Figures.	Names.	Values.
1.....	one.....	one unit.
2.....	two	two units.
3.....	three.....	three units.
4.....	four	four units.
5.....	five	five units.
6.....	six	six units.
7.....	seven	seven units.
8.....	eight	eight units.
9..	nine	nine units.
0.....	nought, or zero....	no units.

3. The first nine of these have a real value in units, and are therefore called *significant* figures. The 0 has no value in itself, but serves to alter the values of the other figures when combined with them. Thus, if a 0 be placed on the right of any of the other figures, it increases the value of the latter tenfold; if two 0s be so placed, the value is increased a hundred-fold, &c. On the same principle the significant figures are combined among themselves. The following are some of the combinations with their values:—

10.... ten.	100.... one hundred
16.... sixteen.	101.... one hundred and one.
20.... twenty.	110.... one hundred and ten.
52.... fifty-two.	209.... two hundred and nine.
60.... sixty.	290.... two hundred and ninety.
79.... seventy-nine.	902.... nine hundred and two.
93.... ninety-three.	603.... six hundred and three.

4. In any number consisting of two or more figures, the right-hand figure is said to occupy the unit's place; the next, the ten's place; the next the hundred's place, &c. This is more fully shown in the following table:—

hundred thousand millions.	ten thousand millions.	thousand millions.	hundred millions.	ten millions.	MILLIONS.	hundred thousands.	ten thousands.	THOUSANDS.	hundreds.	tens.	UNITS.	
BILLIONS.												
3,	2	5	4,	9	1	6,	2	4	7,	1	8	5

The whole of these figures taken together represent—Three billions, two hundred and fifty-four thousand, nine hundred and sixteen millions, two hundred and forty seven thousand, one hundred and eighty-five. In connexion with this table the following points should be noticed:—

1. The fourth figure represents thousands; the seventh, millions; and the thirteenth, billions.

2. Ten tens are one hundred; a thousand thousands are one million; a million millions are one billion.

3. Each figure has its value increased tenfold for every succeeding place it occupies in proceeding from the right to the left. In the number 111, the 1 on the right hand represents one unit, the 1 next to it represents ten units, and the 1 in the third place represents ten times ten, or one hundred units.

4. If the figures in the table be divided by commas into sets of three, the first set represents units; that is, single units, tens of units, and hundreds of units. Similarly, the second set represents thousands; the third, millions; the fourth, thousands of millions, &c. This division is a great assistance in the quick reading of large numbers.

5. The value which a figure has when standing alone, or in the unit's place of a combination, is called its *simple* value; its value in any other place of a combination is called its *local* value. In the numbers 8 and 28, the 8 has its simple value of eight units in both cases, but in the latter number the 2 has a local value of twenty units.

6. In the number—eight millions, five hundred and three thousand and seven; the tens, hundreds, and tens of thousands do not occur. In expressing this and similar numbers in figures, for every such omission a 0 is interposed, to preserve the proper local value of the significant figures. Thus the above number is written, 8,503,007. In like manner the number, sixty millions, nineteen thousand, and twenty, is expressed by the figures 60,019,020.

It must be carefully noticed, that in whole numbers the figure 0 never stands alone, nor in the left-hand place of a number consisting of two or more figures, but is only of service when in the unit's or some intermediate place of a combination. Hence, 30, 200, and 7,060 are proper numbers; but 0, 08, 027, 006, are altogether unmeaning.

Examples. Write in figures—

Ex. 1. Three hundred and twenty-seven. *Ans.* 327.

Here we first write 7 in the unit's place; then, as twenty is two tens, we write 2 in the ten's place; lastly, to express three hundreds, we write 3 in the hundred's place.

Ex. 2. Seven thousand and fifteen. *Ans.* 7015.

As fifteen is one ten and five units, we write 5 in the unit's place, and 1 in the ten's place; then, for no hundreds and seven thousands, we write 0 in the hundred's place, and 7 in the thousand's place.

Ex. 3. Eight millions, three thousand, and sixty-nine.

Ans. 8003069.

Beginning at the right hand, we have nine units, *write* 9; six tens, or sixty, *write* 6; no hundreds, *write* 0; three thousands, *write* 3; no tens of thousands, *write* 0; no hundreds of thousands, *write* 0; eight millions, *write* 8.

Examples. Write in words—

Ex. 4. 13,444. *Ans.* Thirteen thousand, four hundred and forty-four.

Ex. 5. Write in figures and words all the numbers which can be made with the figures 7, 3, and 6, taken together. *Ans.* 736, seven hundred and thirty-six; 763, seven hundred and sixty-three; 376, three hundred and seventy-six; 367, three hundred and sixty-seven; 637, six hundred and thirty-seven; 673, six hundred and seventy-three.

Ex. 6. 47,806,053,400,549. *Ans.* Forty-seven billions, eight hundred and six thousand and fifty-three millions, four hundred thousand, five hundred and forty-nine.

Ex. 1.

First Course.

Write in figures—1. Fourteen. 2. Eighteen. 3. Eighty-one. 4. Ninety-seven. 5. Four hundred and twenty-nine. 6. Two hundred and ninety-four. 7. Eight thousand and five. 8. Five thousand and eighty. 9. Seventy-two thousand and thirty-five. 10. Four hundred and twenty-six thousand, seven hundred and ninety-nine. 11. Fifty thousand and five. 12. Three hundred thousand, three hundred and one. 13. One million and one hundred. 14. Four millions, seventy thousand, and eighty. 15. Six millions and six thousand. 16. Eight hundred thousand and eighty-eight. 17. Nine millions, ninety thousand, and nine hundred. 18. Seven millions, seven hundred thousand, and five hundred. 19. Sixty millions and two. 20. Two millions, four hundred and five thousand, six hundred and one.

Write in words—21. 3117; 22. 4029; 23. 6844; 24. 7209; 25. 67245; 26. 72000; 27. 80340; 28. 70006; 29. 349205; 30. 400028; 31. 649974; 32. 200007; 33. 804066; 34. 6934781; 35. 1010001; 36. 50060000; 37. 6190091; 38. 44406600; 39. 80009400; 40. 81189060.

Second Course.

Write in figures—1. Three hundred and sixty millions. 2. Five hundred and seven millions, sixty thousand, and fourteen. 3. Seven thousand millions, and nine thousand. 4. Fifty-four thousand and seventy millions, six hundred and three thousand, and ninety. 5. Two hundred thousand and one millions, eight hundred and forty thousand, three hundred and twenty-nine. 6. Thirty billions, forty millions, and fifty thousand. 7. Six billions, two thousand millions, and one. 8. Nine hundred and seven billions, four hundred and two thousand millions, eight hundred thousand, and eight hundred.

9. The National Debt is eight hundred and three millions, seven hundred and thirty-three thousand, nine hundred and fifty-eight pounds. Express this in figures.

10. Four millions, four hundred and ninety-five thousand, seven hundred and forty-eight sovereigns were coined at the Mint in the year 1857. Express this number in figures.

Write in words—11. 6493257; 12. 790004003; 13. 800080080; 14. 62000040010; 15. 6008000304059; 16. 10000000001100; 17. 345060000000000.

18. Write in words all the numbers which can be made with the figures 4, 7, and 1, taken together.

19. Write in words all the numbers which can be made with the figures 6, 8, 0, and 0, taken together.

20. The surface of the earth comprises 196,602,223 square miles. Express this in words.

SIMPLE ADDITION.

7. Concrete numbers are of the same *kind* when they represent a common property: they are of the same *denomination* when they are collections of the same unit. The numbers 6 tons and 8 tons are of the same kind, because they represent the common property *weight;* they are of the same denomination, because the unit of each is *one ton.* So, also, 6 tons and 8 pounds are of the same kind, as both represent weight; but they are of *different* denominations, the unit of the former being *one ton,* while the unit of the latter is one *pound.*

8. *Addition* is a method of finding a number which shall contain exactly as many units as two or more given

numbers together. The given numbers are said to be *added together*, and the number found is called the *sum*. In *simple* addition, the numbers to be added together are either all abstract numbers, or all concrete numbers of the same denomination.

The sign + is called *plus*, and when placed between two numbers, signifies that they are to be added together. The sign = is called *equals*, or the *sign of equality;* when placed between numbers, it signifies that they are equal. Thus, 2 + 3 = 5 is read, *two plus three equals five;* that is, if 2 be added to 3, the sum will be 5.

Example 1. John has 4 marbles, Thomas 6, William 3, and Henry 5; how many have they altogether?

The numbers are written in a column, and a line drawn under the lowest figure. Then, beginning at the bottom, we say, *five units and three units are eight units; eight units and six units are fourteen units; fourteen units and four units are eighteen units;* we then write 18, and draw a double line underneath. In practice we say, *five and three are eight, and six are fourteen, and four are eighteen.*

4	
6	
3	
5	
—	
18	Sum.

Example 2. Add together 7425, 6083, 2984, 3669, and 8462.

Arranging the numbers in a column, and drawing a line under the lowest number, we begin at the bottom of the unit's column, and say, *two and nine are eleven, and four are fifteen, and three are eighteen, and five are twenty-three.* Now twenty-three units are three units and two tens; therefore, *we write* 3 *under the unit's column, and add* 2 *to the column of tens.* We then say, *two and six are eight, and six are fourteen, and eight are twenty-two, and eight are thirty, and two are thirty-two.* Now, thirty-two tens are three hundreds and two tens; therefore, *we write* 2 *under the ten's column, and add* 3 *to the line of hundreds.* In like manner we proceed till the last line is added, which we find to be twenty-eight thousands; and as there is no other line to which we can add the 2, we write down the complete number, 28.

7425	
6083	
2984	
3669	
8462	
—	
28623	
232	

If the sum of any line is less than ten, we have no figure to add to the next line; for any sum from 10 to 19, we add 1 to the next line; from 20 to 29, we add 2; from 30 to 39, we add 3, &c. This is called *carrying* 1, 2, 3, &c., to the next line. The small figures under the answer are those we had to carry in example 2.

Example 3. 426 + 349 + 63 + 4806. *Ans.* 5644.

In this example we arrange the numbers in a column, and proceed as in Example 2.

Ex. 2.

First Course.

2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
5	5	7	3	9	2	6	8	2	4	8
4	1	4	7	4	7	4	6	8	9	8
2	8	9	8	1	8	9	5	7	6	7
7	6	2	5	8	5	4	7	7	7	5

14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
7	6	8	4	6	9	3	6	4	9	5
4	3	2	9	1	6	5	2	7	8	8
7	7	7	6	8	4	1	9	2	7	9
6	4	9	8	4	5	7	4	8	8	9
3	2	4	3	5	1	4	5	5	4	7
4	9	3	7	9	8	8	7	3	6	8

26.	27.	28.	29.	30.	31.	32.	33.	34.	35.	36.
24	48	29	62	67	84	19	75	49	54	85
61	27	65	71	50	16	7	60	6	17	96
78	49	48	49	43	33	44	40	72	83	79
53	64	73	58	98	60	60	58	9	19	67
49	81	19	23	67	48	36	13	53	40	88
34	38	65	79	46	25	80	97	4	9	59

.	38.	39.	40.	41.	42.	43.	44.	45.	46.
3	641	419	760	4	647	491	639	476	865
6	497	528	300	6	758	76	396	643	568
5	785	337	500	25	473	408	936	891	586
8	888	746	407	809	314	92	693	352	470
6	373	155	86	47	896	871	369	278	685
3	956	864	448	63	188	718	963	554	856
7	262	673	973	8	931	178	708	169	719
6	519	482	767	907	580	781	870	938	658

7.	48.	49.	50.	51.	52.	53.	54.
32	4257	2763	5342	7182	2635	8491	2861
16	630	4528	4763	1453	2972	6350	3972
81	7108	8771	8231	2916	8159	2870	465
32	3254	293	1792	3271	2643	4058	723
71	67	946	2006	2468	6354	5697	142
46	5224	4819	3047	4020	247	3276	61
09	3276	3254	5004	7300	5	1452	17
53	4829	2716	6392	4606	63	6325	8290

55.	56.	57.	58.	59.	60.	61.	62.
3257	7040	6402	5403	8235	6047	493	8476
4269	8235	8573	6042	2476	8258	7628	4978
7158	2687	4296	89	9568	7766	487	7689
1423	4902	4449	7045	1047	4892	59	5879
6571	6283	3587	649	3566	5693	6732	6384
7321	4956	7402	7865	20	798	4289	9847
8549	5799	6779	4513	452	44	27	6572
2465	8004	4872	8979	9748	895	641	9999

63.	64.	65.	66.	67.	68.	69.
47256	59278	97563	86849	56328	60008	28765
83294	43681	42769	70090	29472	89474	87469
64792	87776	51407	48260	88888	5932	43892
54829	54921	80596	59496	54932	85697	95627
36742	63957	31912	45629	65749	13245	49328
95886	82462	40009	34765	82543	64781	56784
88779	53494	94786	82179	92949	49576	95476
69534	86592	54321	64278	78009	93837	82919

70.	71.	72.	73.	74.	75.	76.
56328	58476	83681	56676	82735	463	84632
49763	32598	94732	82178	46251	8271	98716
8940	47565	20498	38969	54726	598	76849
93	1924	62951	50046	29384	27	53878
876	83491	34782	87930	8742	67248	96789
84	90074	86849	42518	563	937	89978
69475	65320	97638	60948	29	4628	65876
8564	79480	48797	59476	7	586	78958

77. Add together, sixty thousand, four hundred and seventy-nine; eight hundred thousand and fifty; nine thousand, eight hundred and forty-seven; eighty-six; five thousand, nine hundred; and six millions, forty thousand, and seven.

78. Add together, four hundred and seventy-three thousand; 6498; seven millions, and ninety-seven; 84639; sixty-three thousand; and 98476.

79. Add together, 94760; 82753; six hundred thousand, and nine hundred; eighty-six millions; 496352; 798; and 49.

80. Add together, 84793; 69504; 75498; 481; 9; 684723 and 84.

81. 6842 + 7963 + 4821 + 85 + 9604 + 79 + 8 + 63547 + 954.

82. 4930 + 6978 + forty thousand + 8463 + 975628 + 3947.

83. 63 + 742 + 8970 + 59 + 67482 + 8 + 927314 + 628.

84. 9500 + one million + 76543 + 8957 + 62 + 90 + 8479.

Second Course.

1. The area of England is fifty thousand, nine hundred square miles; Scotland, thirty-one thousand; Ireland, thirty-two thousand, five hundred and thirteen. What is the area of the United Kingdom?

2. At a fair there are 4235 horses, 3279 oxen, 54936 sheep, 7741 lambs, and 948 pigs. How many in all?

3. A ship sails from London to New York, a distance of 3219 miles; from New York to Rio Janeiro, 5247 miles; from Rio Janeiro to Cape Town, 4672 miles; from Cape Town to Calcutta, 5678 miles; from Calcutta to Melbourne, 6019 miles. What was the distance from London to Melbourne by the ship's course?

4. The duty on paper for the year 1858 was, in England, £920,609; in Scotland, £271,436; in Ireland, fifty-two thousand, and ninety pounds. What was the whole amount?

5. Add together 6275, 49231, 826307, 94823, and 473089.

6. Find the sum of 846, 92378, 769840, 942607, and 98.

7. What number is equal to the sum of eight thousand, and sixteen; fourteen millions, forty thousand, and seventy; five hundred thousand and sixty-six; eight millions, and five; and 94763208?

8. Add together nine millions, and ninety; eighty thousand, and seventy; sixty thousand, and four hundred; eight millions, eight thousand, eight hundred and eight; and forty millions, four thousand and six.

9. In each of two boxes there are 847694 pins; in each of two other boxes there are double the number. How many in all?

10. Add together 8463907, 3773899, 64637, 99, and 60874.

11. Add together 6 millions, 87 thousand, 9 thousand, 43 hundred, 78 thousand, 4920768, 982 thousand, and 49786309.

12. Of the number of letters which could not be delivered in the year 1857, owing to incorrect or illegible addresses, 1,460,791 were returned to the writers; 102,234 were re-issued with corrected addresses; 196,799 were returned unopened to foreign countries; and 264,253 were destroyed, as the writers could not be discovered. What was the total number?

13. Tom has 79 marbles, William has 48 more than Tom, and Philip has 26 more than Tom and William together. How many have they in all?

14. In the year 1857 there were imported into Great Britain and Ireland 3,437,957 quarters of wheat, 1,710,299 of oats, 1,701,470 of barley, 76,048 of rye, 159,899 of peas, 305,775 of beans, and 1,150,783 of Indian corn. What was the total number of quarters?

15. Add together 478965, 692073, 582967, 934082, 697886, 593410, 684793, 482765, 827934, 692718, and 579897.

16. Find the sum of 847963, 942769, 8593, 64791, 982, 87, 648251, 853496, 98, 6435, and 59267438.

17. Add together six hundred and forty-seven millions, eight hundred and three thousand and nine; three billions, six millions, and eighty thousand; five hundred thousand and ninety-seven millions, eight thousand, and sixty-five; eleven thousand millions, and nineteen thousand; and fourteen billions, seventy-nine thousand millions, six hundred and eight thousand, and eight.

18. Seventy-four millions, and nine, + 635728 + 9476 + 392865 + four hundred thousand and sixty-six + 584793 + 69 + 984 + 57932.

19. In a school there are 47 boys in the first class, 34 in the second class, as many in the third class as in the first and second together, and as many in the fourth class as in the second and third together. How many are there in the four classes?

20. A railway train contains 64 first-class passengers, 59 more second class than first class, and 127 more third class than first and second together. How many are in the train?

21. The revenue of the United Kingdom for the year 1858 was derived as follows:—from the Customs, £23,472,881; from the Excise, £17,731,000; from the Stamp Duty, £7,728,343; from Taxes, £3,136,003; from the Property Tax, £7,852,625; from the Post Office, £3,025,000; from Crown Lands, £276,940; and from miscellaneous sources, £1,938,727. What was the total revenue?

22. There are four boxes of oranges, the first of which contains 648, the second 123 more than the first, the third 49 more than the second, and the fourth twice as many as the first and third together. How many oranges are there?

23. What number is that which exceeds the sum of 923 and 4786 by five thousand and twelve?

24. There are 64725 cannon balls in one pile; another pile contains 400 more than the first; and the number in a third pile exceeds the sum of those in the other two by 29. How many balls are there in the three piles?

25. If a certain number be diminished by 3478, and 546 be taken from the remainder, there will be 56498 left. What is the number?

26. A certain number of boys stand in three rows; in the first row there are 17; in the second row there are twice as many and 5 more; in the third row there are 7 more than in the other two rows together. How many are there in all?

27. What number is equal to the double of 479 + 6809 + 54?

28. Two Greenwich pensioners were conversing one day, when one remarked to the other, "It is just 43 years since you and I were at the battle of Trafalgar; I was then 24 years of age, and you were 5 years older." The battle of Trafalgar was fought in 1805. When did this conversation take place? and how old was each at the time?

SIMPLE SUBTRACTION.

Subtraction is that operation by which we find the ence between two unequal numbers. The greater er is called the *Minuend;* the lesser, the *Subtrahend;* sult is called the *Difference* or *Remainder.*

Simple Subtraction the given numbers are either abstract, or both concrete numbers of the same nination.

sign − which is called *minus*, is employed to indicate the on of subtraction, the greater number being placed before the nd the latter after it. Thus, 9−5=4, is read "nine, minus uals four,"—that is, if 5 be taken from 9, the remainder is 4.

mple 1. Take 472356 from 628198.

te the lesser number under the greater, nder units, tens under tens, &c., and line underneath. Then, beginning at ht hand of the lower line, we say, "6 *leaves* 2;" this 2 we write down. Simi-

628198	minuend.
472356	subtrahend.
155842	remainder.

re say, "5 *from* 9 *leaves* 4," for the next step. At the third e have to take 3 from 1; in order to do this we add 10 to the ing it 11, and say "3 *from* 11 *leaves* 8." Now, as we have 10 hundreds to the one hundred in the upper line, we must add al number to the lower line; and 1 thousand being equal hundreds, we add 1 to that figure in the lower line which nts thousands; that is, we add 1 to 2, making it 3; then say, *m* 8 *leaves* 5." Next, we subtract 7 from 2 in the same way lid 3 from 1, namely, "7 from 12 *leaves* 5." Lastly, we add e 4, and say "5 *from* 6 *leaves* 1." The remainder is 155842, vhich we draw a double line.

process of adding 10 to the top line and 1 to the next figure in ttom line is often called *borrowing* 10 and *carrying* 1. The if this process rests on the following axiom:—*if two unequal s be equally increased, their difference remains the same.*

mple 2. 78429−69347.

his example we arrange the numbers as before, and as follows:—*seven from nine is two, four from is eight, four from four is nought, nine from eighteen seven from seven is nought.* We do not, however, nis last 0, as it has no meaning at the left-hand of a number.

78429
69347
9082

Ex. 3.

First Course.

1.	2.	3.	4.	5.	6.
246345	478256	513876	642015	743526	832047
123121	234104	203435	231012	712224	532014
7.	8.	9.	10.	11.	12.
478314	613025	987654	491728	835604	758496
425101	210022	654321	450213	712201	724243
13.	14.	15.	16.	17.	18.
989898	897728	899999	796894	889977	576789
154456	660532	897653	6352	233223	72244
19.	20.	21.	22.	23.	24.
427305	329476	624075	895647	302946	674007
214615	143528	241038	596478	105692	582009

25. 497562 − 182491
26. 758631 − 248634
27. 497906 − 192569
28. 397621 − 84672
29. 435700 − 347620
30. 876490 − 516004
31. 298763 − 78932
32. 310405 − 61499
33. 492356 − 259764
34. 987829 − 899999
35. 403200 − 63147
36. 625731 − 538746

37. Subtract 8934625 from 49875921; 5296847 from 63492784; 32105792 from 62495718; 8249 from 10000; 5760001 from six millions; 492739658 from ten thousand millions and four thousand.

38. Take three hundred and ninety thousand, and forty, from twenty millions, twenty thousand, and twenty.

39. From seven thousand and two millions, four hundred and five thousand and thirty-three, take six hundred and forty millions, nine hundred thousand and eight hundred.

40. A gentleman has an income of £6040 a-year, and spends £4279. How much does he save?

41. A cattle dealer takes 6472 bullocks to market, and sells five thousand and sixty. How many does he take home?

42. A market woman has 2347 eggs in one basket, and 4026 in another; she sells 5678. How many has she left?

43. What number added to 825 will make 426003?

44. What number added to sixty thousand will make a million?

45. Mount Chimborazo is 21,424 feet high, and Mount Everest, the highest mountain in the world, is 29,000 feet high. What height is Everest above Chimborazo?

46. Take 847635 from the sum of 4725679 and 5897.

47. What number must be added to the sum of 64 and 97 to make 200?

48. What number added to the difference of 39 and 50 will make 100?

Second Course.

1. A bricklayer, who was going to his work on a Wednesday morning, said to a friend, "I built up two thousand and fifty bricks on Monday, I exceeded that number yesterday by forty-five, and I mean to-day to make the whole number up to seven thousand." How many bricks had he to build up that day?

2. Tom and Harry had each 96 marbles; after playing some time Harry lost 42, but he afterwards won 73. How many had each when they left off?

3. A ship sailing due west at the rate of 13 miles an hour passes another sailing in the same direction at the rate of 7 miles an hour. How far are they apart in two hours? and how far would they have been apart had the latter vessel been sailing due east?

4. A farmer was carrying some turnips home in three carts; one cart contained 846; another, 763: when they were all put together, the number was just 347 less than three thousand. How many turnips were brought home by the third cart?

5. A and B commenced business separately with the same capital; A clears a profit of £92 the first year and £117 the second year: B has a profit of £138 the first year, but loses £35 the second year. How much is A worth more than B?

6. Take the sum of 4269 and 5947 from the difference between ten thousand and a million.

7. $6847 + 59321 + 67 - 8243 + 265 - 427$.

8. In the year 1857 there were brought to England 2,964,033 gallons of wine from Portugal, 4,628,790 gallons from Spain, and 796,760 gallons from France. How many more gallons were brought from Spain than from France and Portugal together?

9. Take the difference of a thousand and a million from their sum.

10. Find a number equal to $6479 - 248 + 82651 - 479 - 9237$.

11. John has 9 pence, William 5 pence more than John, and Charles 3 pence more than both together. There are 48 pence in four shillings. Is their whole money more or less than four shillings? and by how much?

12. There are three apple-trees in a garden, on one of which there are 296 apples, on another, 475, and had there been 67 more on the third tree there would just have been a thousand apples in the garden. How many were on the third tree?

13. Halley's Comet is seen once in every 76 years: it was last visible in 1835. In what year was it seen before that, and when will it be seen again?

14. Says Charles to James, "If you give me 6 marbles, I shall have 40, but if I give you 6, we shall each have the same number." How many had each?

15. What number must be added to $847 + 9063 - 4785 + 19 - 839$, to make it five hundred thousand and five?

16. Take 413056278 + 4987 + 889 from twenty billions.

17. From two billions and one million take thirty millions and eight.

18. From six hundred thousand millions take 847 + 9563 + 91 + 49.

19. A farmer took to market 647 bullocks, 941 sheep, and 200 pigs: he sold 498 bullocks, 346 sheep, and all the pigs except 12; he then bought 491 lambs, and exchanged three bullocks for 24 pigs. How many animals did he take home?

20. Take a million except 5 from 4273569 − 845672.

SIMPLE MULTIPLICATION.

10. *Multiplication* is a method of finding a number which shall be equal to another one taken a certain number of times. To multiply 6 by 4, is to find a number which shall be equal to 6 taken 4 times, or four sixes together.

The number to be multiplied is called the *Multiplicand.* The number it is to be multiplied by is called the *Multiplier.* The result of the operation, or the number found, is called the *Product.*

In *Simple Multiplication* the multiplicand is either an abstract number, or a concrete number of one denomination. The multiplier is *always* an abstract number. The product is the same kind of number as the multiplicand.

11. We learn by addition that the sum of two fours is 8, of three fours 12, of four fours 16, of five fours 20, &c. If we treat all numbers from 1 to 12 in this way, and arrange the results in order, we construct the *Multiplication Table.* By committing this table to memory we are enabled to say at once that nine sixes are 54, seven fours are 28, &c., without having to set down nine sixes or seven fours in a column, and to add them together in order to find what their sum is. Multiplication may therefore be said to be a contracted mode of performing addition, when the numbers to be added together are all alike.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The horizontal row of figures along the top, and the vertical column on the left hand, each contain all the numbers from 1 to 12. The second horizontal line consists of the results obtained by multiplying all the numbers from 1 to 12 by 2; the third row contains the results obtained by multiplying by 3; the fourth row by 4, &c. Suppose we wish to know how many *five sevens* are: we begin with the figure 5 in the left-hand column, and proceed horizontally till we come at the number directly under the figure 7 at the top, which in this case is 35. In like manner we find *eight fours* to be 32; *nine nines* to be 81; &c.

In learning the table we begin with the second line from the top, and say—*twice one are two, twice two are four, twice three are six, twice four are eight, twice five are ten*, &c., to *twice twelve are twenty-four*. In like manner we proceed in the third line from *three ones are three*, to *three twelves are thirty-six*. And similarly, through the whole table, to *twelve times twelve are one hundred and forty-four*. The scholar must constantly practise this table till he can promptly and correctly repeat the whole in any order whatever.

The sign × which is called *into*, is employed to indicate multiplication: thus $9 \times 4 = 36$, is read *nine into four equals thirty-six*, or, *nine multiplied by four equals thirty-six*.

Example 1. Multiply 647025 by 6.

Here we arrange the numbers as in the margin, placing the multiplier under the unit's figure of the multiplicand, and drawing a line underneath. We then say *six times five are thirty*, and as 30 is no units and three tens, we *write* 0 and *carry* 3. We have next to multiply two tens, which we do by saying *six times two are twelve*, and adding the 3 tens we had to *carry* gives us 15 tens; we therefore *write* 5 and *carry* 1. In the next step we say *six times nought is nought*, and adding the 1 we had to *carry* gives us 1, which we write down. We have nothing to carry to the next multiplication, but have simply to say *six times seven are forty-two*: we *write* 2 and *carry* 4. And proceeding in the same way we say *six times four are twenty-four, and four are twenty-eight; six times six are thirty-six, and two are thirty-eight.* We then draw a double line underneath.

647025
6
3882150

Example 2. In four baskets there are 3476 apples: three of them each contain 846 apples; how many apples are there in the fourth basket?

If we multiply 846 by 3, we shall get the number of apples in three of the baskets. This we find to be 2538; and by subtracting this number from the whole 3476 apples, we find that there are 938 apples in the fourth basket.

846	3476
3	2538
2538	938

Ex. 4.

1.	2.	3.	4.	5.
462734	570891	689074	547263	827491
2	2	2	3	3
6.	**7.**	**8.**	**9.**	**10.**
826549	974253	742675	820064	574263
4	4	5	5	6

11. 6457291 × 2.
12. 8263427 × 2.
13. 5943681 × 3.
14. 4782009 × 3.
15. 5204678 × 3.
16. 2465703 × 4.
17. 4788642 × 4.
18. 5178209 × 5.
19. 6342587 × 5.
20. 8200506 × 6.
21. 7904087 × 7.
22. 3470256 × 6.
23. 497620354 × 7.
24. 826024761 × 8.
25. 540925638 × 9.
26. 247946087 × 8.
27. 384654123 × 8.
28. 879462815 × 9.

29. Multiply 64723, 54961, and 82795 separately by 2.
30. Multiply 49276, 84325, and 69074 separately by 3.
31. Multiply 82047, 63592, and 80294 separately by 4.
32. Multiply 9623498 and 82007563 each by 5 and by 6.
33. Multiply 8246397 and 431256789 each by 6 and by 7.
34. Multiply 690047038 and 82634958 each by 7 and by 8.

35. Multiply 4256301879 by 3, 4, 5, 6, and 7, separately.
36. Multiply 84702591638 by 4, 7, 8, and 9, separately.
37. Multiply 640235, 894721, and 935462 separately by 9.
38. Multiply 82405768 and 4273516 by 3, by 6, and by 9.
39. Muitiply 842756193 by 2, by 3, by 4, by 5, and by 6.
40. Multiply 9473865109 by 7, by 8, by 3, and by 9.
41. Multiply the sum of 684, 93507, and 4631 by 6.
42. Multiply $8273 + 59 + 456 \times 92738$ by 7.
43. Multiply the difference of 64793 and 5482 by 8.
44. Multiply 6274931—82746 by 4.
45. Multiply the difference between a million and a thousand by 11.
46. A person had twelve cottages built, in each of which there were 34768 bricks. How many bricks were used altogether?
47. Multiply 684793 and 82157496 by 10, by 11, and by 12.
48. Multiply the difference between three hundred and a hundred thousand by 9, by 11, and by 12, separately.
49. Multiply 6470083195 by 8, by 11, by 7, and by 12.
50. Find the product of the sum and difference of 3001 and 2994.

12. We have already learnt that when 6 and 8 are multiplied together they make the product 48. For this reason 6 and 8 are called *factors* of 48, from the Latin word *facio*, which signifies *to make*. We may therefore separate or *resolve* 48 into the two factors 6 and 8, or into the two factors 4 and 12. The figure 1 is not regarded as a factor; consequently, in whole numbers, all factors are greater than unity. A number which can be broken up into factors is called a *composite* number: a number which cannot be broken up into factors is called a *prime* number. 10, 14, 22, 24, and 32 are composite numbers: 7, 11, 13, 17, 19, and 23 are prime numbers

The factors themselves may sometimes be resolved into smaller factors; when this cannot be done they are called *prime factors*. 2 and 7 are prime factors of 14; 3 and 11 are prime factors of 33; but although 4 and 5 are factors of 20, they are not *prime* factors, because the 4 itself may be resolved into the factors 2 and 2. The prime factors of 20 are therefore 2, 2, and 5.

The separation of a number into its factors enables us to multiply by any number found in the multiplication table.

Example. Multiply 47563 by 24.

Here we first multiply by 4, which gives 190252 as the product; we next multiply this product by 6, which gives 1141512 as the final product. As 3 and 8, and also 2 and 12, are likewise factors of 24, we might have multiplied by either of these pairs of factors with the same result.

47563
4
190252 = 4 times 47563
6
1141512 = 4 × 6 or 24 times 47563

In the following oral questions the factors mentioned should be such as are found in the multiplication table.

Oral Questions. What are the factors of 42? of 44? of 54? of 72? How may a number be multiplied by 30? In how many ways can a number be multiplied by 18? by 28? by 48? What are the factors of 81? of 84? of 96? of 120? How may a number be multiplied by 132?

Ex. 5.

1. Multiply 6473058 by 14, by 16, by 18, and by 22.
2. Multiply 4791264 by 24, by 28, by 32, and by 33.
3. Multiply 8419876 by 36, by 20, by 42, and by 30.
4. Multiply 7354987 by 54, by 63, by 80, and by 56.
5. Multiply 629357 and 8426319 each by 64 and by 72.
6. Multiply 82997, 460093, 774683, and 89967 by 77.
7. Multiply 47506309 by 96, by 132, by 88, and by 49.
8. Multiply 64093, 58927, and 826354 separately by 144.
9. Multiply 463928 by 42, and 9135786 by 32, and find the difference between the results.
10. Multiply the sum of 647503 and 894639 by 56, and the difference between a million and thirty thousand by 72, and find the difference between the products.
11. Multiply 67394 by 63, by 24, by 50, by 36, and by 81, and find the sum of all the products.
12. Multiply 8200463 by 6, by 11, by 15, by 84, and by 108, and add all the products together.

13. It has been already stated (See 3) that a number is increased to ten, a hundred, or a thousand times its value by affixing one, two, or three 0s respectively, as the case may be. This method may be in part extended to any case in which the multiplier ends in one or more 0s. Suppose, for example, we are required to multiply

7809 by 40, we may proceed in any one of the lowing ways:—

647809	647809	647809
5	4	40
3239045	2591236	25912360
8	10	
25912360	25912360	

he result is seen to be the same in each case; but it is obvious t the method employed on the right hand of the page, where we e only to multiply by 4 and affix 0 to the product, is the simplest best.

Example. Multiply 36498 by 60, by 90, by 700, and by 3000.

36498	36498	36498	36498
60	90	700	3000
2189880	3284820	25548600	109494000

3. Multiply 649387 by 100, by 1000, by 80, and by 600.
4. Multiply 543070 by 90, by 400, by 50, and by 7000.
5. Multiply 847603 by 60, by 500, and by 8000, and find by how ch the latter product exceeds the sum of the two former.
6. Multiply 84635 by 50000, and 6794865 by 56, and subtract latter product from the former.

Ve may combine this method of multiplication with that of ltiplication by factors, as follows:—

Example. Multiply 469834 by 560, and by 4200.

469834	469834
8	6
3758672	2819004
7	7
263107040	1973302800

re $560 = 56 \times 10 = 8 \times 7 \times 10$, which explains the method employed the first case. And $4200 = 42 \times 100 = 6 \times 7 \times 100$, which explains method of proceeding in the second case.

17. Multiply 847936 by 630, and by 3200.
18. Multiply 3491067 by 4800, and by 2800.
19. Multiply 67948 by 350, by 4900, and by 840.
20. Multiply 794063 by 8100, by 9600, and by 990.

14. When several numbers are multiplied together the result is called the *continued product* of the numbers. *Example.* Find the continued product of 6, 4, and 11. $6 \times 4 \times 11 = 264$ *Answer.*

The *square* of a number is the product obtained by multiplying the number by itself. The square of 8 is 8×8, or 64.

The *cube* of a number is the product of its square and the number itself. The square of 8 is 64; therefore the cube of 8 is 64×8, or 512. The cube of $6 = 6 \times 6 \times 6$, or 216.

15. It will be seen hereafter that the method of multiplying by factors is more useful in compound than in simple multiplication. The following method of multiplying by any number, whether prime or composite, greater than 12, is better adapted to simple multiplication, but cannot be applied to the multiplication of compound numbers.

Example 1. Multiply 48306 by 67.

Since $67 = 7 + 60$, we first multiply by 7, by which we get 338142: we next multiply by 60, and get 2898360, which we place under the product obtained by multiplying by 7: then, adding these results together, we find the whole product to be 3236502. The 6 in the multiplier represents 6 tens; if, in multiplying by this we take care to place the first figure in the ten's column, we may omit the 0 at the end which stands under the 2.

```
 48306
    67
------
338142
2898360
-------
3236502
```

Example 2. Multiply 570436 by 2043.

Here $2043 = 2000 + 40 + 3$, we therefore multiply first by 3, then by 4, and affix a 0, then by 2 and affix three 0s.

```
    570436
      2043
-----------
   1711308  =      3 times top line.
  22817440  =     40 times    "
1140872000  =   2000 times    "
-----------
1165400748  =   2043 times    "
```

by omitting the 0s the working would stand as in margin. The next figure to the left of the 4 in the tiplier is 0; we must therefore put the first figure sined by multiplying by 2 one place more to the left n we should otherwise have to do, that is, under the nstead of under the 4. To avoid mistakes, this it should be carefully noticed: it is a good plan to down the 0s, as in (2) and (3) of the following mples:—

570436	
2043	
1711308	
2281744	
1140872	
1165400748	

(1)	(2)	(3)
4726	62481	49276
325	30450	13008
23630	3124050	394208
9452	249924	14782800
14178	1874430	49276
1535950	1902546450	640982208

ability to mistake occurs in multiplying by 1, in e we are apt to place the 6 under the 0 or 8, instead Proper care will avoid this.

Ex. 6.

79 × 43.	17. 845670 × 76.
83 × 57.	18. 903547 × 85.
75 × 46.	19. 820658 × 90.
79 × 35.	20. 841007 × 00.
36 × 28.	21. 500670 × 88.
29 × 69.	22. 840700 × 45.
28 × 73.	23. 690178 × 08.
46 × 49.	24. 945708 × 07.

by 123, and by 245.
8257639 by 872; 4980007 by 806;
485; 6534927 by 884; 7080000 by

; 6894792 by 703; 8439278 by 856;
548; 9147518 by 600; 6409382 by

509; 7984653 by 486; 8257461 by
1806 by 764; 5894760 by 982; 8003507
8.
by 5490; 64927100 by 6008; 58452304
8090.

30. Multiply 49354927 by 4635; 65920743 by 7804; 48520927 by 7265; and 87964523 by 4916.

31. Multiply 4679328 by 4926; and 8796543 by 8009.

32. Multiply 9464875 by 6475; and 6859328 by 4978.

33. Multiply 5946382 by 8479; and 9874667 by 9875.

34. Multiply 4903068 by 9080; and 9487567 by 8039.

35. Multiply 4987960 by 8764; and 5467910 by 6789.

36. Find the product of 382576004 and 82756; of 463008096 and 46709; and of 893607492 and 50864.

37. Find the product of 826390572 and 64738; of 793804679 and 58190; and of 648793281 and 75009.

38. Find the product of 546078375 and 67182; of 395426978 and 85907; and of 463892034 and 45289.

39. Find the continued product of 647, 231, 905, and 863.

40. Find the continued product of 356, 8279, 46, and 854.

Second Course.

1. There are fifteen carriages attached to a railway train, each of which will accommodate thirty-six passengers. All the carriages are full except one, which contains nineteen persons. How many passengers are in the train?

2. A book contains 192 pages; each page contains 44 lines; and each line contains on an average 52 letters. How many letters are there in the book?

3. Multiply three millions, forty-nine thousand, and sixty-eight by nine hundred and seven thousand, and eighty-four.

4. Find the value of $68479 + 3205 - 4792 + 86324$, and multiply the result by eighty-nine thousand, and sixty-five.

5. A man earns four shillings a-day; one of his sons earns three shillings a-day; and another son earns two shillings a-day. How many shillings will they earn together in 54 days?

6. There are 24 desks in a school; at each desk 7 boys are writing; each boy has to write 5 lines in his copy book. How many lines will be written by the whole school?

7. A earns 18 pence a-day, which is fourpence more than B earns; C earns ninepence a-day more than B. How many pence will they earn together in 8 weeks, working six days a-week?

8. A steamer going at the rate of 16 miles an hour, passes a ship sailing in the same direction at the rate of 11 miles an hour. How far will they be apart in 9 days, reckoning 24 hours a-day? How far would they have been apart if they had been sailing in *opposite* directions?

9. Find the continued product of the square of 13, the cube of 8, and the square of 34.

10. On a railway which is 87 miles long, there are 11 telegraph wires for 59 miles, and 9 wires for the remainder of the distance. What distance would this wire extend if it were single?

11. What number is that which exceeds the product of 847 and 659 by the difference of 86457 and 9798?

12. A train leaves York for London, a distance of 198 miles, at 8 o'clock in the morning, travelling at the rate of 29 miles an hour; another train leaves London for York at 9 o'clock in the morning, travelling at the rate of 37 miles an hour. How far will they be apart at 10 o'clock? how far at 11 o'clock? and how far at 12 o'clock?

13. What number must be added to 896 × 5476 to make ten millions?

14. What number must be taken from the square of 84036 to leave 59407869 for a remainder?

SIMPLE DIVISION.

16. *Division* is an operation by which we find how many times one number is contained in another. The number to be divided is called the *Dividend;* the number we divide by is called the *Divisor:* the result of the operation is called the *Quotient.* The quotient expresses the number of times the dividend contains the divisor.

In *Simple Division* the dividend, divisor, and quotient may be all abstract numbers. When the dividend is a concrete number and the divisor an abstract number, the quotient is like the dividend. When the dividend and divisor are both concrete numbers, the quotient is an abstract number.

The sign ÷ which is called *by*, indicates the operation of division: it is placed between the numbers, and signifies that the number which precedes the sign is to be divided by the one which follows the sign. Thus, 72÷6=12 is read, seventy-two divided by six equals twelve.

We know from the multiplication table that 3 times 4 are 12; we may therefore say that 3 is contained in 12 four times; or that 4 is contained in 12 three times. We also know that 3 added to 7 times 6 will make 45; we may therefore say that 7 is contained in 45 six times and 3 over, or, that 6 is contained in 45 seven times and 3 over.

Example 1. Divide 846305 by 5.

Make a curve on the left of the dividend; on the left of this curve place the divisor; then draw a line under the dividend, and proceed as follows:—5 is contained in 8 once, and 3 over—*put down* 1 *and carry* 3; 5 is contained in 34 six times

Divisor 5)846305 dividend.
169261 quotient.

and 4 over—*put down* 6 *and carry* 4; 5 is contained in 46 nine tim and 1 over—*put down* 9 *and carry* 1; 5 is contained in 13 twice ar 3 over—*put down* 2 *and carry* 3; 5 is contained in 30 six times—*p down* 6; 5 is contained in 5 nce—*put down* 1. The quotient 169261, under which we draw adouble line.

Example 2. 647853÷8.

We arrange the numbers as in Example 1, and proceeding in the same way, we find that the last division is—8 is contained in 13 once and 5 over.

8)647853
80981 +

This 5 is called the *remainder*, and is separated from the quotient b the sign +.

A number which can be divided by 2 without a remainder is calle an *even* number: a number which cannot be so divided is called a *odd* number. All numbers ending in 0, 2, 4, 6, or 8 are even num bers: those ending in any other figure are odd numbers.

The proper meaning of the remainder in a division will be explaine hereafter.

Ex. 7.

1. 462084÷2
2. 527312÷2
3. 629478÷2
4. 943570÷2
5. 843116÷2
6. 774590÷2
7. 963500÷2
8. 471354÷3
9. 526317÷3
10. 827985÷3
11. 114771÷3
12. 257691÷3
13. 154763÷2
14. 546296÷4
15. 947380÷4
16. 790468÷4
17. 427565÷5
18. 394170÷5
19. 826015÷5
20. 471064÷3
21. 820953÷4
22. 627131÷5
23. 415674÷6
24. 739014÷6
25. 325494÷6
26. 524014÷4
27. 627953÷5
28. 413624÷4
29. 730218÷6
30. 514395÷7
31. 273056÷7
32. 397432÷7
33. 926352÷8
34. 394256÷8
35. 276544÷8
36. 507066÷7
37. 394209÷9
38. 563427÷9
39. 825048÷9
40. 325467÷4
41. 294632÷5
42. 827040÷3
43. 256439÷6
44. 475622÷7
45. 374113÷8
46. 256914÷9
47. 804636÷7
48. 294003÷9

49. Divide 34295628 by 2, by 3, by 4, by 5, and by 6.
50. Divide 54627836 by 4, by 5, by 3, by 7, and by 8.
51. Divide 82467918 by 6, by 7, by 4, by 9, and by 8.
52. Divide 11114076 by 3, by 5, by 9, by 7, and by 4.
53. Divide 60405312 by 9, by 10, by 8, by 6, and by 11.
54. Divide 82760391 by 3, by 7, by 11, by 9, and by 12.
55. Divide 68004728 by 4, by 9, by 12, by 8, and by 11.
56. Divide 21539952 by 7 by 4, by 8, by 6, and by 9.
57. Divide 26937360 by 8, by 5, by 6, by 10, and by 9.
58. Divide 23348556 by 6, by 7, by 9, by 11, and by 12.
59. Divide 4263545 by 4, by 6, by 5, by 11, and by 9.

60. Divide 49070116 by 9, by 11, by 12, by 8, and by 6.

61. Add together 34796, 5204, 69783, 42937, 8064, and 625479, and divide the sum by 3, by 5, by 7, and by 9.

62. Subtract 9463829 from 42798364, and divide the remainder by 8, by 6, by 5, by 9, by 11, and by 12.

63. Multiply 82346 by 59, and divide the product by 7.

64. Multiply 65479 by 504, and divide the product by 8 and by 7.

65. Multiply 39476 by 8316, and divide the product by 9, by 11, by 6, by 8, by 5, and by 12.

66. Subtract 94763285 from 104206391, and divide the remainder by 4, by 5, by 6, by 9, by 11, and by 8.

67. Add together 39462, 295, 8604, 95371, 67, 9482, 639578, and 94269, and divide the sum by 4, by 8, by 5, by 7, and by 9.

68. Multiply thirty-seven thousand, and sixty-eight, by three thousand, seven hundred and eighty, and divide the product by 7, by 4, and by 9.

17. We may divide by any number found in the multiplication table by separating it into its factors, and dividing by these in succession.

Examples 1.—Divide 689808 by 28.

Since $28 = 7 \times 4$, we first divide 689808 by 7, which gives the quotient 98544; we then divide 98544 by 4, which gives 24636 as the final quotient or answer. The same result would have been obtained had we divided by 4 first and then by 7.

7)689808÷28
4)98544
24636

2.
8)375410÷48
6)46926÷2
7821 rem. 2

3.
7)230587÷56
8)32941
4117÷5 rem. $= 5 \times 7 = 35$

4.
9)386397÷54
6)42933
7155÷27

5.
8)230587÷56
7)28823÷3
4117÷4, rem. $= 4 \times 8 + 3 = 32 + 3 = 35$.

6.
3,0)64794,2÷30
21598÷2

7.
4)427397÷32
8)106849÷1
13356 rem. 5.

8.
7,00)62734,00÷700
8962

9.
6,0)82635,4÷60
13771÷34

10.
3,00)41256,49÷300
13752÷149

To find the actual remainder in any case. (1) When there is a remainder in dividing by the first factor only, we have simply to bring that remainder down to the same line as the answer, as in Ex. 2. (2) When there is a remainder in the second division, but none in the

first, we multiply that remainder by the first divisor, as in Exs. 3 and 4. (3) When there is a remainder in dividing by both factors, we multiply the last remainder by the first divisor, and add the first remainder to the product, as in Exs. 5 and 7.

In dividing by any number ending in one or more cyphers, we mark off those cyphers and also the same number of figures from the right of the dividend, and divide by the remaining figure of the divisor; the remainder (if any) in the division, together with the figures cut off from the dividend, is the actual remainder. See Exs. 6, 8, 9, and 10.

Ex. 8.

1. Divide 3205888 by 16; 13051728 by 18; 11345235 by 15; 11051922 by 14; 17772489 by 21; and 99565400 by 22.
2. Divide 14267225 by 25; 20207424 by 24; 27123936 by 32; 237412242 by 27; 428560 by 20; and 15152976 by 36.
3. Divide 15115960 by 40; 89104530 by 30; 48826700 by 50; 13027344 by 33; 24025364 by 44; and 6279434 by 49.
4. Divide 14091315 by 35; 37285752 by 42; 15349248 by 48; 29583135 by 45; 10933160 by 28; and 6429764 by 44.
5. Divide 13305114 by 54; 56367108 by 63; 69135780 by 70; 8936950 by 55; 18247200 by 96; and a million by 64.
6. Divide 27545112 by 72; 12533304 by 84; 83920023 by 99; 23763102 by 66; 37882264 by 56; and 6492513 by 35.
7. Divide 40532338 by 77; 51349920 by 120; 8496500 by 100; 56108052 by 132; 101959319 by 121; and 6204054 by 54.
8. Subtract 14393287 from 49563281, and divide the remainder by 108. Divide thirty-six millions by 72.
9. Add together 9478265, 49327, 946, 2158, 674932, 45054498, and 847926, and divide the sum by 132.
10. Multiply 371946 by 56, and divide the product by 72.
11. Multiply 8146767 by 7128, and divide the product by 88.
12. Divide the sum of 426357, 94218, 629473, 851196, 492387, 64927, 898, and 16848 by 144.
13. Divide 6094173 by 24, by 42, by 45, and by 56.
14. Divide 4107368 by 49, by 72, by 36, and by 48.
15. Divide 24007104 by 32, by 66, by 48, and by 81.
16. Divide 14562072 by 28, by 54, by 121, and by 72.

18. We may divide by any number greater than 12 as follows:—

Ex. 1. 3051381 ÷ 47.	Ex. 2. 49321784÷638.
47)3051381(64923 *Ans.*	638)49321784(77306, rem. 556.
282	4466
231	4661
188	4466
433	1957
423	1914
108	4384
94	3828
141	556 remainder.
141	

In Ex. 1, the first two figures of the dividend are 30, but as we cannot divide 30 by 47, we take the first three figures of the dividend, namely, 305. We then multiply 47 by a number such that the product shall be less than 305, and when subtracted from 305 shall leave a remainder less than 47. The number in this case is 6, which we place to the right of the dividend, as the first figure of the quotient. The product is 282, which we place under 305, and draw a line underneath. We then subtract 282 from 305, which leaves a remainder 23: this completes the first step. We then bring down the figure 1 from the dividend and place it on the right of the remainder 23, making 231. This 231 is divided in the same manner as 305 was; the quotient figure being 4, the product 188, and remainder 43. The next figure of the dividend is then brought down. In like manner the process is performed till all the figures of the dividend are brought down.

On bringing down the figure 8 in Ex. 2, we see that the number 438 is less than the divisor 638. When this is the case, we put a 0 in the quotient and bring down the next figure of the dividend. Practice alone will give facility in fixing on the right quotient figure in any case.

Ex. 9.

First Course.

1. Divide 5542563 by 13; 8740924 by 17; 191944 by 23; 10395584 by 19; 9229946 by 34; and 18226829 by 37.
2. Divide 23739358 by 46; 47446384 by 59; 45092384 by 71; 18064558 by 86; and 12256842 by 39.
3. Divide 140433684 by 42, by 91, by 56, and by 39.
4. Divide 35758160 by 57, by 48, by 95, and by 76.
5. Divide 36517634 by 47; 9217404 by 63; and 4743466 by 74.

6. Divide 14543278 by 59; 53966883 by 87; 69564465 by 93; 26470453 by 79; 22279520 by 68; and 50433836 by 98.

7. Divide 114270912 by 54, by 72, by 98, and by 84.

8. Divide 133519698 by 246; 229343400 by 468; 35769082 by 349; 14270068 by 627; and 659740284 by 714.

9. Divide 248302356 by 716; 170568480 by 408; 341795663 by 631; 64901073 by 826; and 184961416 by 569.

10. Divide 983033688 by 637, by 294, by 273, and by 392.

11. Multiply 640093 by 507, and divide the product by 403.

12. Take 31375721 from 514639721, and divide the remainder by 839.

13. Add together 64739, 42916, 56817, 41392, 68977, 74329, 9264, and 897, and divide the sum by 497.

14. Divide 64913780 by 4932, by 6407, by 8900, and by 4516.

15. Divide 140087997 by 3291; 516103518 by 6258; 266620200 by 8756; 264398121 by 4839; and 365450371 by 6743.

16. Divide 202245120 by 2565, by 1881, by 1296, and by 1539.

17. Divide 64835904 by 847, by 6234, by 96, and by 4978.

18. Divide 33761667156 by 54073, and by 208124.

19. Divide 403255548712 by 765804, and by 40506.

20. Divide three hundred and three thousand eight hundred and fifty-five millions, four thousand, four hundred and four, by one million, eight hundred and seventy-three thousand, one hundred and sixteen.

19. The following relations between the divisor, dividend, quotient, and remainder should be noticed:—

1. Quotient × divisor = dividend.
2. Dividend ÷ quotient = divisor.

When there is a remainder.

3. (Quotient × divisor) + remainder = dividend.
4. (Dividend − remainder) ÷ divisor = quotient.
5. (Dividend − remainder) ÷ quotient = divisor.

The operations indicated by the signs within the brackets should be performed first.

Second Course.

1. The quotient in a division is 641579, and the divisor 4058, what is the dividend?

2. By what number must 8479 be multiplied to give 110227 for a product?

3. A spelling book contains 56 pages; on each page there are four columns; and in each column there are 48 words. How many days will it take a boy to finish the book at the rate of 46 words a-day, supposing him to have done 33 pages already?

4. A train leaves London for Aberdeen, a distance of 559 miles, travelling at the rate of 34 miles an hour; in how many hours will it be exactly 83 miles from its destination?

5. What is the least number which can be taken from a million, so that the remainder may be exactly divisible by 467?

6. A box contains 34 pounds of candles, there being 4 candles in each pound: had there been five pounds more, the whole box would just have lasted 52 nights. How many candles were burnt each night?

7. What is the least number which must be *added to* twenty millions, so that the sum may be divided by 89 without a remainder?

8. A merchant buys a cargo of sugar for £7579, and clears £7 profit on every £143 spent in the purchase. What is his total profit?

9. Suppose, in the last question, the merchant had bought the sugar for £8883, and sold it for £9450, what profit would he have cleared on every £141 he spent in the purchase?

10. John has 34 marbles, but after receiving 11 from Tom, and giving 3 to Harry, he has three times as many as each. How many had each at first?

11. A certain product is 53525287340, and one of the factors is 826: what is the other factor?

12. A Government train leaves London for Liverpool, a distance of 210 miles, at six o'clock in the morning, travelling at the rate of 20 miles an hour: an express train leaves London for the same place at nine o'clock in the morning, travelling at the rate of 35 miles an hour. At what distance from Liverpool will the fast train overtake the other?

13. A, B, and C enter into partnership; A invests £4236; B £5493; and C £3631. They make a profit of £3 on every £40 invested: what is their total profit?

14. A has 41 nuts, B has 12, C has 29, and D has 14. They wish to divide them equally: how must this be done?

15. A school is divided into five classes: in the first class there are 24 scholars, which exceeds the number in the second class by 7: there are four more in the third class than in the second; three times as many in the second, third, and fourth classes together as there are in the second; and as many in the third and fifth together as there are in the first and second together. How many more scholars must be admitted to the school before it can be equally divided into six classes?

16. John says to Thomas, "I am twice as old as you;" "and," says Thomas to William, "if I were four years older, I should be twice as old as you;" their grandfather remarked "that his age was double of all theirs together." This was in 1858, William being then 6 years old. In what year was each born, beginning with the oldest?

17. Multiply 47056238946 by 74139568, and divide the product by the difference between 40536781 and 40534765.

18. Multiply $648+5192-3279+89476-91479$ by 427613, and divide the product by 84006.

19. In a division the divisor is 806, and had the dividend been

increased by 123, the quotient would have been the same as the divisor, leaving no remainder. Find the dividend and remainder.

20. What is the least number which must be added to a million, so that when the sum is divided by 46, the remainder may be 1?

*MISCELLANEOUS APPLICATIONS.**

20. When one number is a factor of two or more others, it is called a *common factor* of these numbers. 9 is a common factor of 27, 45, and 63.

When several numbers having common factors are connected by the signs of multiplication and division, the operation may be shortened by striking out those factors.

Ex. 1. $24 \div 28 \times 42 \times 72 \div 27 \div 16$.

Those numbers which are divisors are written below the line; multipliers are written above the line; the sign × is placed between the numbers. The second form is derived from the first by dividing 24 and 16 each by 8, 42 and 28 by 7, and 72 and 27 by 9. The third form is derived from the second by dividing 3 and 3 each by 3, 6 and 2 by 2, and 8 and 4 by 4. The fourth form is obtained from the third by multiplying all the upper numbers together and all the lower numbers together. Finally, dividing 6 by 1 the answer is 6.

$$\frac{24 \times 42 \times 72}{28 \times 27 \times 16} = \frac{3 \times 6 \times 8}{4 \times 3 \times 2} = \frac{1 \times 3 \times 2}{1 \times 1 \times 1} = \frac{6}{1} = 6 \text{ } \textit{Ans.}$$

In practice a line is drawn through those numbers which are divided by a common factor, and the new numbers written above or below, as follows:—

Ex. 2. Divide the continued product of 16, 40, 33, 25, and 24 by the product of the square of 5 and the cube of 8.

$$\frac{\overset{2}{\cancel{16}} \cdot \overset{5}{\cancel{40}} \cdot 33 \cdot \overset{5}{\cancel{25}} \cdot \overset{3}{\cancel{24}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{8}} \cdot \underset{1}{\cancel{8}} \cdot \underset{1}{\cancel{8}}} = 990 \textit{ Ans.}$$

The 1s are often omitted, and a point is frequently used instead of the sign ×. It is obvious that we must always, in dividing by the common factor, divide one of the upper and one of the lower numbers together. This operation is termed *cancelling*.

21. The signs × and ÷ do not affect any number separated from them by either of the signs + or −, unless that number be enclosed in brackets.

* The pupil may omit the questions under the miscellaneous applications during his first course, and proceed to Reduction.

Exs. (1). $4+2\times3=4+6=10$; (2). $(4+2)\times3=6\times3=18$;
(3). $27-6\times3=27-18=9$; (4). $(27-6)\times3=21\times3=63$;
(5). $\{(4-2)\times(4+3)+6\}+(12-7)=(2\times7+6)+5=20+5=4$.

*22. The quotient obtained by dividing a number by 2, 3, 4, or 5, is called the *half, third, fourth,* or *fifth* part accordingly.

To find two-thirds of a number, we divide by three which gives one-third, and then multiply by 2. To multiply a number by a mixed multiplier, such as five and a-half, or six and three-fourths, we multiply by the whole number first, and then find the fractional part as already explained, and add it to the product.

One-half, two-thirds, five and a-half, &c., are written, $\frac{1}{2}$, $\frac{2}{3}$, $5\frac{1}{2}$, &c. These are called fractions, and will be more fully explained hereafter,

Examples. Multiply 62412 by $\frac{1}{4}$, by $5\frac{1}{4}$, and by $30\frac{1}{4}$.

(1). 4)62412
15603 = one quarter.
3
46809 = three quarters of 62412.

Or we may multiply and then divide thus:—

62412
3
4)187236
Ans. 46809

(2). 62412
5
312060 = 5 times 62412.
31206 = half of 62412.
343266 = $5\frac{1}{2}$ times 62412.

(3). 62412
30
1872360 = 62412 × 30.
15603 = 62412 ÷ 4.
1887963 = 62412 × $30\frac{1}{4}$

23. To find the average of two, three, four, or five, &c., numbers, we add them together, and divide the sum by 2, 3, 4, or 5 accordingly.

Examples. 1. In a school there are present 89 children on Monday, 93 on Tuesday and Wednesday, 91 on Thursday, and 84 on Friday. What was the average attendance for the week?

2. Suppose, in the last question, there had been 90 children present on Friday, what would have been the average attendance?

3. A railway train consists of 2 first class and seven second class carriages, and contains 37 first class, and 116 second class passengers. What is the average number of passengers in each carriage?

* This article, especially Exs. 2 and 3, should be studied in connexion with the reduction by multiplication of Cloth, Lineal, and Square Measure.

(1). 89 (2). 89 (3). 2 + 7 = 9 = total no. of carriages.
93 93 37 + 116 = 153 = total no. of pass ngers.
93 93
91 91 9)153
84 90 *Ans.* 17 = average no. in a carriage.
5)450 5)456
90 *Ans.* 91⅕ *Ans.*

In questions on average the remainder, if any, should be written as in Ex. 2.

Ex. 10.

1. Multiply the half of 6894 by the third of 41706.
2. Find the value of 42 × 27 ÷ 84 × 60 ÷ 45 × 79.
3. Multiply the square of 96 by the cube of 33, and divide the product by the square of 132.
4. Divide 78 nuts between A and B, so that the one may receive twice as many as the other.
5. A farmer bought a horse and cow for £39, but the horse cost twice as much as the cow?—what was the price of each?
6. Multiply 64270 by 5½, and 7241608 by 2¼.
7. Add three-eighths of the square of 40 to two-thirds of the cube of 42.
8. 63 ÷ 108 ÷ 84 × 72 ÷ 88 × 48 × 56 × 77.
9. 694 + 47 × 8 − 432 ÷ 8 − 1 + 479 + 325 ÷ 13 + 1.
10. From two-thirds of 11114070 take the square of 678.
11. What number must be added to four-fifths of ten thousand to make it equal to seven-eighths of a million?
12. A gentleman gives 80 pence between A, B, and C; for every penny which C receives, B receives threepence, and A sixpence. How much did each receive out of the 80 pence?
13. A person bought a pound each of tea, coffee, and sugar for 78 pence: the coffee cost three times as much as the sugar, and the tea was three times the price of the coffee. What was the price of each per pound?
14. The attendance in a school was 123 on Monday, 125 on Tuesday and Wednesday, 125 on Thursday, and 122 on Friday. What was the average attendance for the week? and what would it have been if the attendance had been 2 more on Tuesday, and 1 less on Friday?

REDUCTION.

24. *Reduction* is an operation by which we are enabled to change the denomination in which any concrete number is expressed, without altering its value.

Reduction is effected by multiplication, or by division, or by both combined.

Reduction by Multiplication.

Examples. Reduce (1) £34 17s. 0¾d. to farthings.

2. Reduce 43 lbs. 9 oz. 13 dwts. 23 grs. to grains.

(1.) £	s.	d.	(2) lbs.	oz.	dwts.	grs.	(3) lbs.	oz.	dwts.	grs.
34	17	0¾	43	9	13	23	43	9	13	23
20			12				12			
680	s.		525	oz.			525			
adding 17	s.		20				20			
697	s.		10513	dwts.			10513			
12			24				24			
8364	d.		42075				42055			
4			21026				21028			
33456	f.		252335	grains.			252335	grains.		
adding										
33459	farthings.									

Explanation. Ex. 1. Since £1 = 20s., we convert £34 into 680s. by multiplying by 20; to this we add the 17s. in the given number, making 697s. Next, we convert 697s. into pence by multiplying by 12, since 12d. = 1s. For a similar reason we reduce the pence to farthings, by multiplying by 4. The addition of 17s. and of 3f. in the course of the operation should be carefully noticed.

Ex. 2. The table for Troy weight explains the reason for employing 12, 20, and 24 as multipliers. Instead of making a separate addition for the oz., &c., as in Ex. 1, these quantities are added in the process of multiplication. For example, we say, "12 times 3 are 36 *and* 9 *are* 45;" and similarly in multiplying by 20 and by 24.

In reducing the dwts. to grs. in Ex. 2, we have to add 23 grs. from the given number. In the first form of working the question, the whole 23 grs. are added in multiplying by 4; in the second form, the 3 units of grs. only are added in multiplying by 4, and the two tens of grs. are added in multiplying by 2. Either form may be employed.

Reduction by Division.

3. Reduce 33459 farthings to pounds, shillings, &c.
4. Reduce 890547 ounces to tons, cwts., qrs., &c.

4	33459		16	4	890547
12	8364	−3f.		4	222636 − 3 oz.
2,0	69,7	−0d.	28	7	55659
	34	−17s.		4	7951 − 2 } 23lbs.
				4	1987 − 3 }
				2,0	49,6 − qrs.
					24 −16 cwt.

Ans. £34 17s. 0¾d.

Ans. 24 tons, 16 cwt., 3 qrs., 23 lbs., 3 oz.

Ex. 3 is the converse of Ex. 1, and either method may be used to prove the correctness of the other. In Ex. 4, the divisors 16, 28, 4, and 20 are derived from the table of Avoirdupois weight. Any remainder after a division is of the same denomination as the dividend in that division. In dividing by 28, in Ex. 4, there is a remainder 2 in dividing by the factor 7, and 3 in dividing by the factor 4; this 3 is multiplied by 7 and the 2 added to the product, making the total remainder 23 lbs. For the mode of dealing with such remainder see (17).

25. In the tables for Cloth, Lineal, and Square measure, the fractional numbers $2\frac{1}{4}$, $5\frac{1}{2}$, and $30\frac{1}{4}$, occur. The following examples will explain the method of reducing by these numbers.

5. Reduce 14 Eng. ells, 3 qrs., 3 na., 1 in. to inches.
6. Reduce 6 m. 5 fur. 33 po. 4 yds. to yards, &c.

ells.	qrs.	na.	in.		m.	fur.	po.	yds.
14	3	3	1		6	5	33	4
5					8			
73					53			
4					40			
295					2153			
$2\frac{1}{4}$					$5\frac{1}{2}$			
591					10769			
73 − 3 quarter inches					1076 − 1 half-yard = 18 in.			

Ans. $664\frac{3}{4}$ inches.

Ans. 11845 yds. 1 ft. 6 in.

See 22, Exs. 2 and 3, for multiplying by $5\frac{1}{2}$, &c.

7. Reduce 641209 in. to miles, furlongs, &c.
3. Reduce 623413 sq. ft. to acres, roods, &c.

641209

53434 — 1 in.

17811 — 1 ft.
2

35622

323,8 — 4 half yds. = 2 yds.

80 — 38 po.

10 — 0 fur.

(3.)
9)623413

69268 — 1 sq. ft.
4

121 { 11 | 277072
11 | 25188 — 4 } 103 qr. yds.
4,0 | 228,9 — 9 } = 25¾ yds.
4 | 57 — 9 poles.

14 ac. — 1 rood.

r. 10 m. 38 po. 2 yds. 1 ft. 1 in. *Ans.* 14 ac. 1 ro. 9 po. 25 yds. [7 ft. 108 in.

n Ex. 7 we have to divide 17811 yds. by 5½; this is most conveniently done by bringing the 17811 yds. into 35622 half yds., and n, since there are 11 half yds. in 5½ yds., we divide 35622 by 11. a similar reason we convert yds. into quarter yds. in Ex. 8, and de by 121, which is the number of quarter yds in 30¼ yds. The a sq. yard. is converted into feet by multiplying the 3 by 9, and ding the product by 4; this gives 6¾ ft., and adding the 1 ft. in second line makes 7 ft., the remaining ¾ ft. being multiplied by to convert it into inches.

. Reduce 1 cwt. 1 lb. to Troy weight.

cwt. qrs. lbs.
1 0 1
4
4 qrs.
28
33
8
113 lbs.
7000
791000 grains.

24 { 4 | 791000
6 | 197750

2,0 | 3295,8 — 2 × 4 = 8 grs.

12 | 1647 — 18 dwt.

137 — 3 oz.

Ans. 137 lbs. 3 ozs. 18 dwt. 8 grs.

Ve have in this example the operations of multiplication and sion combined.

Ex. 11.

Reduce—

1. £321 to shillings; £42 to pence; and £17 to farthings.
2. £479 16s. to shillings; 18s. 11d. to pence; and 14s. 9d. to farthings.
3. £27 9s. 4d. to pence; and £28 13s 8¼d. to farthings.
4. £32 18s,; £246 14s. 7d.; and £57 19s. 11½d. to half-pence.
5. 19s. 9¼d.; £28 6s. 10½d.; and £378 9s. 4¾d. to farthings.
6. £427 13s.; and £241 17s. 6d. to sixpences.
7. £96 0s. 8d.; and £634 15s. 4d. to fourpenny pieces.
8. 324 guineas to half-pence; and 648 guineas to florins.
9. £987 to half-crowns; to florins; and to threepenny pieces.
10. £5292 to guineas; to half-guineas; and to half-crowns.
11. 6498 half-guineas to sixpences; and to fourpenny pieces.
12. 17280 pence to £; to half-crowns; and to half-guineas.
13. 849673 farthings to £, &c.; and 64297 half-pence to £, &c.
14. 5 cwt. to oz.; 34 tons to quarters; and 3 qrs. 17 lbs. to drams.
15. 44 tons 15 cwt. 3 qrs. to oz.; and 19 cwt. 2 qrs. 23 lbs. to drams.
16. 623 tons 17 cwt. 1 qr. 27 lbs. 13 oz. 9 drs. to drams.
17. 1709120 qrs. to tons; and 516096 drs. to cwts.
18. 104006317 drs. to tons, &c.; and 69417 farthings to £, &c.
19. 54 tons 13 lbs. to grains; and 3276 fourpences to half-guineas.
20. 317 lbs. Troy; and 5 cwt. 17 lbs. Avoirdupois each to grains.
21. 248 lbs. 6 oz. 19 dwts. 23 grs. to grains; and £97 to mils.
22. 79 lbs. 9 oz. 3 drs. 2 scr. 13 grs. to grains; and 32769 mils to £, &c.
23. 498 lbs. Avoirdupois to Troy weight; and 981047 grs. to lbs., &c., Troy.
24. 1 ton to Apothecaries' weight; and 96000 farthings to mils.
25. 17 yds. to inches; 23 miles to poles; and 2 leagues to yds.
26. 3 m. 7 fur. 34 po. 3 yds. 2ft. 9 in. to inches.
27. A million inches to yds., &c.; and 1 cwt. to lbs., &c., Troy.
28. 49 English ells to nails; and 1056 Fr. ells to yds.
29. An acre to sq. inches; and 3 ro. 37 po. to sq. feet.
30. 27 ac. 2 ro. 29 po. 17 yds. to sq. feet; and 37 lbs. 5 drs. to scruples.
31. 319 cub. yds. to inches; and 50012307 cub. in. to yds., &c.
32. 873 gals. to quarts; 63 qrs. to pecks; and 17 cwt. to Troy weight.
33. 85 qrs. 7 bus. 3 pks. to gallons; and 37 Fl. ells to nails.
34. 10 pipes to gills; 349 barrels to pints; and 697 bush. to quarts.
35. 697 wks. to hours; 27 days to minutes; and 3 wks. to seconds.
36. 6104735 sec. to wks., &c.; 5 qrs. 6 bus. 1 gal. to quarts.
37. 27 wks. 5 d. 19 hrs. 37 sec. to seconds; 24607 dwts. to Avoirdupois weight.

38. 298 ac. 3 ro. 31 po. 7 ft. 119 in. to inches; 34 Fl. ells to Eng. ells, &c.

39. 33 hhds. to pints; 3400771 farthings to £, &c.; 61235 scr. to Troy weight.

40. 914768 pints to barrels; 63457102 gills to puncheons, &c.

*Second Course.**

1. 26 Fr. ells 3 qrs. to inches; and 47 ac. 29 po. to yards.

2. 24552 in. to fathoms; 6973 crowns to mils, and also to £, &c.

3. How many minutes are there from the beginning of March to the end of April? How many seconds in the last three months of the year? How many hours in May and June together?

4. The exportation of beer for 1857 was 434981 barrels: how many half-pints?

5. Add together 34 gross and 79 score. How many sheets are there in 63 reams and 17 quires?

6. How many seconds are there from 9 o'clock a.m., January 17, to 11 o'clock p.m., on the last day but one in November, in leap year?

7. Reduce 6235790 linear inches to miles, &c.; and 30047210612 square inches to acres, &c.

8. A year contains 365 days 5 hours 48 minutes 47 seconds: how many seconds have elapsed from the beginning of 1758 to the end of 1858.

9. Reckoning a year as 365 days, how many years, days, &c., will it take to count a billion, at the rate of 100 a minute?

10. A's property is worth a million farthings; B's is worth 30053 half-crowns; C's is worth 3019 half-guineas. Find the total value in £, &c.

11. 18524 million ounces of water flow over the Falls of Niagara in a minute. How many tons, &c., in the year of 365 days?

12. The length of a side of a square field is 2147 inches: how many acres, &c., does it contain?

* From this point the scholars should proceed with the *First Course* under each set of exercises, returning from time to time to the *Second Course* for the purpose of examination.

COMPOUND ADDITION.

26. In *Compound Addition* the numbers to be added together are of the same kind, and of more than one denomination.

Examples. 1. Add together £13. 4s. 6d., £29. 16s. 4½d., £37. 12s. 9d., £14. 0s. 11½d., and £54. 19s. 4¾d.

2. Find the sum of 4 tons 2 cwt. 3 qrs. 9 lbs., 41 tons 12 cwt. 13 oz., 9 tons 19 lbs., 17 cwt. 3 qrs. 11 oz., and 19 cwt. 27 lbs.

3. Add together 3 m. 7 fur. 22 po. 3 yds., 14 m. 37 po. 5 yds., 26 m. 29 po., 6 fur. 4 yds., and 2 m. 6 fur. 3 yds.

(1) £	s.	d.	(2) tns.	cwt.	qr.	lb.	oz.	3) m.	fur.	po.	yds.
13	4	6	4	2	3	9	0	3	7	22	3
29	16	4½	41	12	0	0	13	14	0	37	5
37	12	9	9	0	0	19	0	26	0	29	0
14	0	11¼		17	3	0	11		6	0	4
54	19	4¾		19	0	27	0	2	6	0	3
£149	13	11¼	56	12	0	0	8	47	5	10	4

The numbers are arranged as in the above examples, so that those of the same denomination may be in the same column. The sum of the first addition in Example 1 is 6 farthings, which equal 1½d.; we therefore put down ½d. and *carry* 1d. to the pence column. The sum in the pence column is 35d., which = 2s. 11d.; we therefore put down 11d., and *carry* 2s. to the shillings column. In the same manner we proceed with each example, always reducing the sum of any column to the next higher denomination, putting down the remainder after the reduction, and *carrying* the number belonging to the higher denomination to its proper column.

Ex. 12.

Add together—

1. £6. 4s. 5d., £9. 7s. 8d., £12. 9s. 6d., and £8. 6s. 8d.
2. £14. 9s. 10d., £16. 13s. 2d., £19. 1s. 4d., and £11. 8s. 9d.
3. £17. 12s. 7d., £24. 14s. 11d., £47. 19s. 9d., and £68. 17s. 8d.
4. £53. 0s. 10d., 12s. 11d., £63. 17s., and £49. 19s. 9d.
5. £89. 11s., £27. 6s. 4d., 19s. 8d., and £76. 14s. 7d.
6. £8. 16s. 4½d., £9. 4s. 8½d., £7. 9s. 6½d., and £5. 7s. 9½d.
7. £9. 4s. 6½d., £8. 6s. 8½d., £9. 9s. 9½d., and £6. 4s. 11½d.
8. £16. 0s. 4½d., £22. 9s. 5½d., £36. 19s. 10½d., and £8. 3s. 6½d.
9. £24. 17s. 11½d., £8. 9s. 6½d., £24. 2s. 11d., and £88. 18s. 0½d.
10. £46. 16s. 10½d., £54. 19s. 11½d., £6. 16s. 9½d., and £37. 17s. 11½d.
11. £9. 0s. 1½d., £24. 16s., £98. 17s., and £46. 17s. 11½d.
12. £83. 13s. 4d., £97. 14s., 19s. 11½d., and 17s. 8½d.
13. 28 lbs. 8 oz. 17 dwt. 23 grs., 9 lbs. 3 oz. 5 dwt. 1 gr., 6 lbs. 5 oz. 7 dwt. 8 gr., 29 lb. 11 oz. 19 dwt. 19 gr., and 13 lb. 8 oz. 15 dwt. 21 gr.
14. 9 lbs. 7 oz. 7 drs. 2 scr. 16 gr., 6 lbs. 1 scr. 12 grs., 7 oz. 2 scr. 5 grs., 15 lbs. 5 oz. 4 dr. 1 scr. 19 gr., and 24 lbs. 6 drs. 13 gr.
15. 1 ton 17 cwt. 3 qrs. 13 lbs., 24 tons 5 cwt. 1 qr. 25 lbs., 11 cwt. 19 lbs., 17 tons 2 qrs. 27 lbs., and 9 tons 18 cwt. 1 qr. 21 lbs.
16. £16. 17s. 5½d., £19. 17s. 8½d., £24. 6s. 9d., £37. 18s. 0½d., £54. 13s. 8½d., £9. 8s. 6d., and £47. 16s. 11½d.

17. 3 lbs. 19 dwt. 4 grs., 11 oz. 17 grs., 16 lbs. 14 dwt., 9 lbs. 4 grs., 10 oz. 16 dwt. 23 grs., 5 lbs. 7 oz., and 22 lbs. 9 oz. 4 dwt. 12 grs.

18. 4 ac. 3 ro. 29 po., 42 ac. 1 ro. 37 po., 9 ac. 2 ro., 63 ac. 33 po., 5 ac. 9 po., 26 ac. 3 ro. 25 po., and 19 ac. 2 ro. 38 po.

19. £67. 4s. 6½d., £83. 9s. 9¾d., £94. 19s. 8½d., 16s. 6d., £8. 0s. 9½d., 17s. 7d., 8s. 8½d., and £49. 17s. 11¾d.

20. 6 m. 7 fur. 19 po. 4 yds., 18 fur. 5 yds., and 3 m. 4 fur. 6 po. 2 yds.

21. 8 wks. 5 dys. 19 hrs. 49 min. 32 sec., 7 wks. 6 dys. 21 hrs. 55 sec., 19 wks. 17 hrs. 47 min., and 57 wks. 1 dy. 23 hrs. 59 min. 59 sec.

22. £19. 7s. 6½d., £43. 8s. 8d., £61. 18s. 3¾d., 11s. 6½d., 19s. 11½d., £4. 6s. 9d., 18s. 6d., 12s. 10d., and £29. 0s. 5½d.

23. 9 tons 27 lbs., 13 cwt. 3 qrs. 5 lbs., 6 tons 7 cwt. 15 oz., 1 qr. 19 lbs., 28 tons 2 qrs. 12 oz., 15 cwt. 27 lbs., 1 cwt. 1 qr. 9 oz.; and 7 tons 19 lbs.

24. 9 qrs. 6 bush. 3 pks. 1 gal. 3 qts., 4 qrs. 5 bus. 2 pks. 1 gal. 2 qts., 7 bus. 1 qt., 12 qrs. 1 pk., 6 qrs. 1 bus. 1 gal., and 4 qrs. 2 pks. 3 qts.

Second Course.

1. Add together £9. 6s. 8½d., £18. 17s. 4d., £27. 8s. 6½d., 19s. 11d., 12s. 2¾d., £36. 17s. 10½d., 13s. 6d., 15s. 8¾d., 8s. 11½d., £43. 0s. 0½d., £67. 18s., £4. 14s. 9d., and £59 16s. 7½d.

2. Add together £34. 14s. 5½d., £69. 17s., £24. 5s. 9d., 6s. 11½d., £43. 16s. 8½d., £29. 19s., £13. 17s. 4d., 18s. 9½d., £65. 8s. 8d., £103. 0s. 11½d., 13s. 10¾d., 19s. 7d., 14s. 0½d., £26. 19s. 6d., £49. 17s., £4. 2s. 5d., 16s. 11d., and 13s. 7½d.

3. Find the sum of £41. 7s. 0¾d., 7s. 4d., £3. 18s. 7d., 19s. 11d., £68. 14s., £12. 9s. 9½d., 13s. 10d., £84. 19s. 8½d., £639. 17s. 2d., 15s. 8d., 18s. 7d., 12s. 6½d., £437. 0s. 10½d., £9. 16s., £4. 1s. 11d., £9. 3s. 5d., 16s. 10½d., and 2s. 4¾d.

4. Add together 14s. 4½d., 9s. 6d., £24. 15s. 3½d., 16s. 11¾d., £346. 8s. 2d., £46. 16s. 8d., £9. 17s., £3. 2s. 6d., 18s. 8½d., £95. 14s. 9d., £27. 5s., £58. 18s. 10½d., 16s. 3d., 4s. 10½d., £27. 10s., £76. 19s. 8¾d., 17s. 9½d., and £38. 16s. 6½d.

5. Find the sum of 2s. 9d., 5s. 4d., 8s. 6½d., 12s. 0½d., 9½d., 16s. 8d., 13s. 4¾d., 8s. 9d., 17s. 11d., 4s. 5½d., 13s. 9½d., 18s. 2d., 2s. 4d., 9s. 7d., and £4. 18s. 0½d.

6. Add together £43. 19s. 6½d., 19s. 2½d., 4s. 6d., 8s. 8d., £287. 6s. 2¾d., £94. 14s., £305. 18s., £9. 0s. 0½d., 15s. 6d., £43. 9s. 8d., 16s. 1½d., 19s. 8½d., £67. 8s. 9d., £4. 12s. 5d., 8s. 11d., 15s. 10d., 17s. 7½d., £53. 13s., £117. 4s., 12s. 4d., 18s. 10½d., 9s. 1½d., £46. 19s. 10d., and £92. 7s. 5d.

7. A farm consists of eight fields, whose areas are as follows:— 9 ac. 3 ro. 27 po., 16 ac. 35 po., 8 ac. 2 ro. 27 yds., 18 ac. 5 po. 26 yds., 12 ac. 1 ro. 18 po. 29 yds., 9 ac. 34 po. 27 yds., 4 ac. 3 ro. 12 yds., and 14 ac. 3 ro. 25 po. What is the total area of the farm?

8. Find the sum of 1000 half-guineas, 5236 pence, 2013 half-crowns, 93 crowns, 62149 farthings, 93 fourpenny-pieces, 617 threepenny-pieces, 917 sixpences, 317 guineas, and £27. 16s. 4 d.

9. Add together £1374. 9s. 6d., £417. 15s. 8¼d., £947. 16s. 11d., £6948. 1s. 10d., £49. 14s. 8½d., 18s. 6d., £325. 11s. 7d., £83. 18s. 2d., £9. 17s., £468. 9s. 1d., £98. 4s. 6d., and £427. 16s. 4d.

10. Twelve workmen are employed in building a house. The first earns £1. 2s. 6d. a-week; the second earns 2½d. a-day more than the first; the third 2½d. more than the second; the fourth 2½d. more than the third, &c. What is the sum of their weekly earnings?

11. What sum is equal to 1000 of each of the gold, silver and copper coins at present current?

12. From what sum must 3471 of each of the current silver coin be taken, to leave a sum equal to 13711 of each of the current copper coin?

COMPOUND SUBTRACTION.

27. *Compound Subtraction* is so called from the numbers with which it deals being of more than one denomination.

Examples. 1. Subtract £15. 16s. 9d. from £37. 4s. 2¼d.

£	s.	d.		£	s.	d.
37	4	2¼		37	24	14¼
15	16	9¼	adding 20s. 12d. 4f. adding £1. 1s. 1d.	16	17	10¼
21	7	4¾d. *Ans.*		21	7	4¾

2. From 17 tons 3 cwt. 9 lbs., take 8 tons 17 cwt. 1 qr. 8 oz.

tns.	cwt.	qrs.	lbs.	oz.		tns.	cwt.	qrs.	lbs.	oz.
17	3	0	9	0	=	16	22	4	8	16
8	17	1	0	8		8	17	1	0	8
8	5	3	8	8		8	5	3	8	8

The arrangement on the left in each of the above examples shows the form in which questions under this rule are to be worked; that on the right explains the process. In Ex. 1, we cannot take ¼d. from ¼d., we therefore add 4 farthings to the ¼d. in the upper line, and then say, ¼ *from* ¼d. *leaves* ¼d. In the next step we add 1d. to the 9d. in the lower line, but as we cannot take 10d. from 2d., we add 12d. to the upper line, and say, 10d. *from* 14d. *leaves* 4d. And similarly with the shillings. It will be seen that the

same sum is added to both the upper and lower line, since 20s. 12d. 4f. = £1. 1s. 1.

In Ex. 2, the upper line is simply changed in form, without any addition, thus—17 tons 3 cwt. = 16 tons 23 cwt.; 23 cwt. 0 qrs. = 22 cwt. 4 qrs.; 9 lbs. 0 oz. = 8 lbs. 16 oz. Either method may be adopted in any case; so also in simple subtraction.

Ex. 13.

Subtract—

1. £4. 2s. 6d. from £9. 3s. 11d.; £8. 5s. 7½d. from £9. 14s. 8¾d.
2. £6. 17s. 11d. from £22. 19s. 11½d.; £4. 9s. 6d. from £5. 17s. 4d.
3. £8. 7s. from £15. 4s. 2d.; £16. 7s. 1d. from £22. 8s.
4. £17. 19s. 6½d. — £9. 14s. 5½d.; £24. 16s. 9d. — £20. 18s. 0½d.
5. £49. 15s. 2½d. — £39. 19s. 3d.; £86. 7s. 1d. — £29. 19s. 10d.
6. £61. — £5. 17s. 4d.; £31. 11s. — 17s. 6d.; £9. 12s. — £8. 13s. 0½d.
7. £81. 14s. 8¾d. — £57. 14s. 11½d.; £93. 5s. 10d. — £76. 18s. 11½d.
8. From 8 lb. 9 oz. 17 dwt. take 4 lbs. 10 oz. 3 dwt. 14 grs.
9. From 37 wks. 4 dys. 19 hrs. 47 min. take 28 wks. 6 dys. 23 hrs. 51 sec.
10. From £116. 13s. 10½d. take £79. 17s. 11¾d.; also 7 bus. 3 pks. from 9 qrs.
11. From £314. 12s. 5½d. take each of the following sums separately: — £4. 17s., £38. 4s. 9d., £43. 15s. 4¾d., £305. 8s. 6¾d., £176. 13s. 6½d., £98. 12s. 11d., £216. 18s. 8½d., and £87. 17s. 10¾d.
12. Find the difference between £17. 9s. 1d., and £42. 6s. 0½d.; 18 lbs. 7 oz. 5 drs. 1 scr. 9 grs., and 9 lbs. 7 drs. 18 grs.; 31 ac. 16 po., and 3 ac. 2 ro. 27 po.; £41. 14s. 9d., and £26. 18s. 6½d.
13. From £513. 3s. 2½d. take the following sums separately:— £417. 9s. 8d.; £93. 15s.; £69. 14s. 11½d.; £507. 11s. 4¾d.; £389. 19s. 7½d.; and £247. 15s. 8½d.
14. Take £1000 from 1000 guineas, and 1000 half-crowns from 700 half-guineas.
15. Take half-a-million fourpenny pieces from ninety thousand sovereigns.
16. From £1435. 9s. 4½d. subtract the following sums separately:— £1. 19s. 7½d.; £468. 16s. 0¾d.; £985. 13s. 9½d.; £1176. 15s. 11d.; £875. 9s. 4¾d.; £66. 9s. 8d.; £478. 18s. 10½d.; and £1000. 0s. 0¾d.

Second Course.

1. Find the difference between the sum of the first six and last seven of the following sums;—£13. 9s. 4½d., £48. 16s., 19s. 8½d., £28. 10s. 9½d., £83. 15s., £187. 0s. 8¾d., 7s. 7½d., 14s. 11d., £96. 13s., £148. 6s. 9d., 19s. 6½d., 16s. 7d., and £50.
2. Find the difference between the amounts of the first ten and last three sums in the last question.
3. What sum must be added to £81. 14s. 0½d. to equal £100.

4. By how much does a million of farthings exceed 3241s. ?

5. What sum must be added to the whole amount in the first question, to make it just £8. 4s. 7½d. less than £1000 ?

6. Which is greater, 96741 half-pence, or £201. 10s. 10½d. ?

7. By how much does a billion inches exceed 62479 miles ?

8. Find the difference between 94127 half-guineas and 614792d.

9. Take 1000 lbs. Troy from 999 lbs. Avoirdupois.

10. From £387. 4s. 9d. + 493 farthings + 67 half-crowns + 117 florins + 6211 fourpenny-pieces, take £42. 9s. + 6214d. + 3101s. + 17s. 6d. + 10000 farthings.

11. Take the sum of 1113 of each of the current copper coin from the sum of 573 of each of the current silver coin, and express the remainder in pounds, shillings, &c.

12. Take the sum of 16s. 9d., 18s. 4½d., £3. 2s., 11s. 5½d., £32., £4. 3s. 11d., 19s. 8¾d., and £8. 15s. 4d., from the sum of 4s. 1d., £34. 8s. 6d., £49. 19s., 1s. 8½d., 16s. 11d., 17s. 0¼d., £3. 12s., £43. 6s. 2½d., 5s. 9d., and 18s. 7½d.

COMPOUND MULTIPLICATION.

28. In Compound Multiplication, the number to be multiplied is of more than one denomination.

Examples (1). Multiply £29. 7s. 6½d. by 7. (2). 15 cwt. 9 lbs. by 11. (3). Multiply 5 ac. 19 po. 11 yds. by 18.

(1). £ s. d.
29 7 6½
7
205 12 7¾ *Ans.*

(2). tons. cwt. qrs. lbs.
15 0 9
11
8 5 3 15 *Ans.*

(3). ac. ro. po. yds.
5 0 19 11
3 × 6 = 18
15 1 18 2¾
6
92 0 28 16½ *Ans.*

(4). Multiply £3. 5s. 2½d. by 67 and 517, separately.

(1). £ s. d.
3 5 2½
8 × 8 + 3 = 67
26 1 8
8
208 13 4 = top line × 64
9 15 7½ = „ × 3
218 8 11½ = „ 67

(2). £ s. d.
3 5 2½
10
32 12 1 = £3. 5s. 2½d × 10
10
326 0 10 = „ × 100
5
1630 4 2 = „ × 500
32 12 1 = „ × 10
22 16 5½ = „ × 7
1685 12 8½ = „ × 517

In Ex. 1, we say "seven farthings make one penny and three farthings; *put down* ¾ *and carry* 1." Next, "7 times 6 are 42, and 1 are 43; 43d. make 3 shillings and 7 pence, *put down* 7 *and carry* 3," &c. In a similar manner all the other examples are worked; the reduction, if necessary, being performed at each separate multiplication, and the number expressing the next higher denomination carried to the multiplication of that denomination.

The arrangement of the multipliers in Exs. 3 and 4 will sufficiently explain the process in each case.

Ex. 14.

1. Multiply 5s. 4d. separately by 3, 5, 7, 11, and 10.
2. Multiply 7s. 9½d. separately by 6, 9, 4, 12, and 7.
3. Multiply £5. 4s. 11d. separately by 2, 5, 9, 7, and 11.
4. Multiply £8. 13s. 7d. separately by 8, 12, 3, 7, and 10.
5. Multiply £12. 16s. 3¾d. separately by 3, 5, 9, 6, and 11.
6. Take 9 times £5. 17s. 3½d. from 7 times £13. 18s. 8¾d.
7. Multiply £4. 3s. 9½d. by 3, by 8, by 11, by 5, and by 12, and add the sum of all the products.
8. Multiply the sum of 17s. 9½d.; £4. 8s. 6¾d.; £13. 8s.; 29. 16s. 11d.; and £47. 9s. 4½d., by 7 and by 12.
9. Multiply £4. 8s. 7d. by 14, 15, 32, and 63, separately.
10. Multiply £47. 13s. 9d. separately by 42, 25, and 54.
11. Multiply £49. 17s. 1½d. separately by 36, 56, and 72.
12. Multiply 3 cwt. 3 qrs. 11lbs. separately by 8, 18, and 28.
13. Multiply 7 wks. 5 days 23 hrs. 17 m. by 6, by 11, and by 56.
14. Multiply £4. 17s. 3½d., and 2 lbs. 1 scr. 7 grs. each by 81
15. Multiply £9. 13s. 5d. separately by 29, 47, and 83.
16. Multiply £11. 17s. 2¾d. separately by 103, 94, and 89.
17. Multiply £5. 14s. 7½d. separately by 157, 213, and 341.
18. Multiply 3 lbs. 5 oz. 13 dwt. by 48, 91, and 134.
19. What is the cost of 793 sheep at £2. 5s. 9d. each?
20. What is the total weight of 79 bales of cotton, each weighing 1 cwt. 1 qr. 21 lbs.

Second Course.

1. Multiply the difference between 149 half-guineas and 517 half-crowns by 319.
2. Reduce 1 ton 3 qrs. to Troy weight, and multiply the result by 69.
3. A boy earns 1s. 7½d. a-day, and his father twice as much; how much do they earn together in 187 days?
4. A earns £1. 7s. 3d. a-week, B twice as much as A, C thrice as much as B, and D four times as much as C. What will they earn together in 11 weeks?

5. In a railway train there are 59 first class and 138 second c passengers; the first class fare is 3½d., and the second class 2½d. mile. What sum would be received for the tickets, supposing first class passengers travelled on the average 95 miles, and the sec class 74 miles?

6. Multiply the difference between 31041 farthings and 79 guin by 349.

7. A person's income is 15s. 9d. per day, and he spends on average 12s. 4d. per day: how much does he save in 20 ye reckoning 365 days to one year?

8. A farmer at a fair sold 127 sheep at £2. 16s. 6d. each; lambs at £1. 5s. 4d. each; and 53 bullocks at 11 guineas each. afterwards bought 9 cows at £10. 17s. each; and 15 horses, for of which he paid 42 guineas each, and for the remainder £35. each. By how much did his sales exceed his purchases?

9. A person buys 17 yds. of cloth at 13s. 6d. a-yard; a dozen h at 1s. 2½d. each; 25 yds. calico at 7½d. per yd.; and 43 yds. damask at 3s. 1½d. per yd. Find the amount of the bill.

10. Find the value of 3247 tons of pig iron at £5. 16s. 6d. a ton

11. Add together £62. 4s. 6d.; £19. 0s. 7½d.; £89. 14s. 6½ £7. 0s. 11½d.; and £26. 18s. 6½d., and multiply the sum by 84.

12. Subtract £47. 18s. 4½d. from £211. 6s. 3½d., and multiply difference by 67.

13. What is the cost of 7 hogsheads of sugar, each weighing 9 c 2 qrs., at 3½d. per lb.?

14. A gentleman's income is £1000 a-year, and he spends £2.14s.9 a-day. How much does he save in a year?

15. A tobacconist retails tobacco at 4s. 9d. per lb. which cost h £21. 2s. 4d. per cwt. What does he gain on a cwt.?

16. A workman's wages are 5s. 6d. per day, and he spen £1. 5s. 10d. a-week. How much can he save in 39 weeks, supposi he is absent 11 days, for which he receives no payment?

COMPOUND DIVISION.

29. In *Compound Division* the dividend is a concre number of more than one denomination.

Two kinds of questions occur in compound division. (1). We ma for example, have to divide £6. 7s. 1d. by 5, that is, we have to find t fifth part of £6. 7s. 1d. The answer to this question is *so much mone* namely, £1. 5s. 5d. In all such questions the answer is of the sam kind as the dividend. (2). We may be required to divide £6. 7s. 1 *by* £1. 5s. 5d., or, in other words, to find *what part* £1. 5s. 5d. is

l.; or, to vary the question still further, to find *how many* . 5s. 5d. is contained in £6. 7s. 1d. The answer to this s the abstract number 5. So in all similar cases.

Divide £29. 16s. 5¼d. by 9.

e say, "9 is contained in 29 three times and *ut down* 3. The £2. that is over we reckon lings, and adding the 16 shillings, we have gs; then "9 is contained in 56 six times and *ut down* 6: next, 2 shillings are 24 pence, e 29 pence; "9 is contained in 29 three times and 2 over," 3: lastly, 2 pence make 8 farthings, and adding the 1 in the dividend, we have 9 farthings, which divided by 9

£	s.	d.	
9)	29	16	5¼
Ans.	3	6	3¼

. 3. Divide £1477. 12s. 3d. by 28 and by 252.

£	s.	d.	
1477	12	3	
)369	8	0¾	
52	15	5¼	*Ans.*

£	s.	d.				
252)1477	12	3(5	17	3¼		
1260						
217						
20						
4352(17						
252						
1832						
1764						
68						
12						
819(3						
756						
63						
4						
252(1						
252						

lanation given above, together with od of dividing by factors in simple will suffice for Ex. 2. In (3) the of the successive remainders to the er denomination is shown in the tion by 20, 12, and 4.

Divide £36254. 3s. 4d. by 1000.

e mark off three figures by a comma, the re- figures, 36, being the the quotient. We then y 20 and 12, and mark igures after each mul- . In dividing by 100 ff two figures, three in

£	s.	d.	£	s.	d.	
1000)36,254	3	4(36	5	1	*Ans.*	
20						
5,083						
12						
1,000						

dividing by 1000, four for 10000; and generally, as many figures are pointed off as there are *noughts* in the divisor. We divide by 3000, 7000, or 15000 in the same way, and then divide the quotient by 3, 7, or 15 accordingly.

Ex. 5. 19 tons 2 cwt. 1 qr. 7 lbs. ÷ 7. Ex. 6. 44 wks. 9 hrs. 12 m. ÷ 8.

	tons.	cwt.	qr.	lbs.		wks.	d.	hrs.	m.
Ex. 5. 7)19	2	1	7	Ex. 6. 8)44	0	9	12		
2	14	2	13 *Ans.*	5	3	13	9 *Ans.*		

Ex. 7. Divide £345. 3s. 9d. by £5. 9s. 7d.

£	s.	d.	£	s.	d.		
345	3	9	5	9	7	1315)82845(63 *Ans.*	
20			20			7890	
6903			109			3945	
12			12			3945	
82845			1315				

For questions like Ex. 7 we have the following

RULE. *Reduce the divisor and dividend to the same denomination, and divide the latter by the former.*

Ex. 15.

1. £34. 10s. 2d. ÷ 2
2. £42. 18s. 9d. ÷ 3
3. £41. 6s. 4d. ÷ 4
4. £97. 0s. 10d. ÷ 5
5. £58. 4s. 6d. ÷ 6
6. £94. 9s. 5d. ÷ 7
7. £19. 15s. 3d. ÷ 9
8. £62. 17s. 4d. ÷ 8
9. 11 cwt. 3 qrs. 1 lb. ÷ 3
10. 19 lbs. 11 oz. 4 dwt. ÷ 4.
11. £43. 19s. 6d. ÷ 6
12. 41 wks. 3 days ÷ 10.
13. Divide £36. 15s. by 6, 4, 7, and 10 separately.
14. Divide £25. 10s. by 8, 3, 5, and 12 separately.
15. Divide £49. 5s. by 5, 9, 12, and 8 separately.
16. Divide £506. 18s. 5¼d. by 7, 9, 5, and 11 separately.
17. Divide £411. 12s. by 21, 24, 56, and 63 separately.
18. Divide £1337. 17s. 6d. by 40, 88, 48, and 132 separately.
19. Divide 444 ac. 24 po. by 28, 72, 63, and 54 separately.
20. Divide £1068. 7s. 6d. by 99, 35, 56, and 84 separately.
21. Divide £1372. 9s. 0¾d. separately by 23, 19, 45, and 67.
22. Divide £5589. 13s. 6d. separately by 49, 26, 78, and 54.
23. Divide £330. 15s. 4½d. separately by 234, 598, and 767.
24. Divide £2890. 12s. 6d. and £15801. 0s. 10d. each by 1000.
25. Divide £69200 separately by 500, 3000, and 8000.
26. Divide 4 tons 1 qr. 21 lbs. separately by 264, 702, and 968.
27. £238. 4s. 4½d. ÷ 1s. 7½d.; £125. 18s. 6d. ÷ £5. 9s. 6d.
28. £74. 17s. 4½d. ÷ £2. 5s. 4½d.; £1000 ÷ £5. 4s. 2d.
29. How often is 3s. 1½d. contained in 500 half-crowns.

ivide 204 lbs. 1 oz. 6 drms. 15 grs. by 5 lbs. 9 oz. 7 drs. 2 scr.

3594. 17s. 9½d. + £67. 19s. 9½d.; 1 m. 1 fur. + 2 ft. 9 in.
me waste land, consisting of 4906 ac. 155 po., is to be divided
tments containing 13 ac. 1 ro. 31 po. each. How many will
?

Second Course.

w many times must £13. 0s. 0½d. be taken to make
9s. 5½d. ?
d together £741. 9s. 4d.; £33. 8s. 7d.; £468. 19s.; 11s. 2½d.;
. 1½d.; £639. 6s. 2d.; £1907. 0s. 11½d.; £87. 16s. 8d.;
3s. 9½d.; and £877. 11s. 1½d.; and divide the sum by
7¼d.
person buys 23 yds. of carpet at 2s. 11d per yard, and a
umber of yards of cloth at 18s. 4d. per yd.; for the carpet
h together he pays £48. 5s. 5d. How many yards of cloth
buy ?
roperty tax of 7½d. in the pound amounts to £259. 17s. 6d.:
the value of the property ?
e rents in a parish amount to £15015, and the poor's rate
to £641. 5s. 3¾d. What is the rate in the pound ?
ver table spoons weighing 5 ozs. 8 dwts. 8 grs. each, cost
id. each: what is the value of the spoons made out of 37 lbs.
silver ?
e fore-wheel of a carriage makes 5 revolutions while the hind
akes 3; and the hind wheel makes 528 revolutions in a mile
lf. What is the circumference of each ?
e fare on a railway is 2¼d. per mile, and the sum realized by
n in travelling 97 miles is £118. 4s. 4½d. What was the
of passengers, supposing each travelled the whole distance ?
140 francs are worth £5. 16s. 8d., how many francs ought to
in exchange for £1. 12s. 6d. ?
merchant buys sugar at 3½d. per lb., and sells it at £1. 19s. 8d.
What does he gain on a pound of sugar ?
butcher bought 3 lambs, weighing 4½ stones each, at 5s. 7½d.
e, and gained 9s. 6¾d. in retailing. What was the selling
lb., reckoning 8 lbs. per stone ?
Then corn is selling at 6s. 3d. per cental, what should be the
a bushel weighing 70 lbs. ?

BILLS.

ril 6, 1859, Henry Williams buys of Charles Wilson the fol-
rticles:—4 lbs. tea at 4s. 2d.; a stone of sugar at 6½d. per lb.;

5 lbs. coffee at 1s. 10d.; 9 lbs. rice at 5½d.; 2 lbs. cocoa at 2s.; ¼ lb. pepper at 1½d. per oz.; 1 lb. mustard at 1s. 8d.; and 9 lbs. cheese at 8½d. Make out the bill.

London, April 6, 1859.

Mr. Henry Williams.

Charles Wilson,

Grocer, Cheesemonger, &c.,

24 & 25, King William Street, City.

E.C.

	£	s.	d.
4 lbs. Tea at 4s. 2d. . .		16	8
1 st. Sugar at 6½d. . .		7	7
5 lbs. Coffee at 1s. 10d. .		9	2
9 lbs. Rice at 5½d. . . .		4	7½
2 lbs. Cocoa at 2s. . . .		4	
¼ lb. Pepper at 1½d. per oz.			6
1 lb. Mustard at 1s. 8d. .		1	8
9 lbs. Cheese at 8½d. . .		6	4½
£	2	10	7

30. When the whole of the articles included in a bill have not been purchased on the same day, the separate dates are entered in the column on the left. When a bill is paid, the receiver of the money writes his name at the bottom, with the word "Received" before it. If the amount of a bill be £2 or upwards, a receipt stamp is affixed, and the person receiving the money writes his name or initials over the stamp.

Make out the following bills as above, supplying, when necessary, names and dates.

Ex. 16.

1. March 20, 1859. Thomas Smith buys of Robert Fox—6 lbs. currants at 10½d.; 8 lbs. raisins at 6½d.; a box of figs weighing 11 lbs.

at 9d.; 3 lbs. tea at 4s. 8d.; 5 lbs. coffee at 1s. 11d.; and 12 lbs. loaf sugar at 9½d.

2. March 4, 1859. William Thomson sells to John Redmond—4 yds. calico at 7½d.; 15 yds. cotton print at 8½d.; 17 yds. flannel at 1s. 11d.; 14 yds. silk at 3s. 4d.; ½ doz. handkerchiefs at 2s. 9d. each; 9 pair socks at 1s. 4d. per pair.

3. 4 lbs. tea at 3s. 10d.; 2 st. sugar at 5d. per lb.; 6 lbs. coffee at 1s. 6d.; 7 lbs. sago at 6½d.; 9 lbs. bacon at 9½d.; 8 lbs. cheese at 7d.; and 5 lbs. butter at 1s. 5d.

4. 8 lbs. coffee at 1s. 7d.; 2 lbs. tea at 4s. 4d.; 3 lbs. cocoa at 1s. 10d.; 1 lb. chocolate at 2s. 2d.; 10 lbs. loaf sugar at 8½d.; 4 jars marmalade at 7½d.; and ½ lb. isinglass at 16s. 6d. per lb.

5. January 3, 1859. Thomas Potts buys of Frederick Harrison—3½ yds. superfine cloth at 17s. 6d.; 7 yds. doeskin at 3s. 11d.; a dozen and a half cambric handkerchiefs at £1. 2s. 6d. per doz.; ½ doz. shirts at 6s. 9d. each; and 18 collars at 9s. per doz. Account paid.

6. Mrs. Waller buys of Ellis & Everington—42 yds. merino at 1s. 9d.; 3 silk dresses at £2. 14s. 6d.; 2 parasols at 11s. 9d.; 29 yds. French muslin at 1s. 3d.; and 33 yds. damask at 3s. 5d.

7. On March 1, 1856, a wine merchant supplies a gentleman with 6 doz. port at £2. 4s. 6d.; 9 doz. sherry at £2. 12s.; and 5 doz. claret at £3. 3s. On June 3, 3 doz. port at £2. 8s.; 5½ doz. Burgundy at £3 15s. On September 11, 4 gals. brandy at 23s.; 2 doz. champagne at 84s.; and 7 doz. sherry at £2. 18s. Account paid, December 30.

8. A bookseller supplies the following—Macaulay's History, 7 vols., at 6s. each; Wordsworth's Poems, 6 vols., at 7s. 6d.; Webster's Dictionary, 15s.; 4 manuscript books at 2s. 9d.; 10 quires note paper at 5½d.; 6 quires cream laid post at 8½d.; 200 envelopes at 1s. 8d. per 100; 3 drawing copies at 1s. 9d., 2s. 6d., and 4s. 8d.; and 10 drawing pencils at 2½d. each.

9. 7 lbs. tea at 4s. 2d.; 3 lbs. chocolate at 2s. 1d.; 5 lbs. coffee at 1s. 9d.; 11 lbs. sugar at 6½d.; 9 lbs. bacon at 9½d.; 23 lbs. cheese at 8½d.; and 5 lbs. butter at 1s. 5d.

10. 11 yds. silk velvet at 12s. 9d.; 6 yds. Brussels net at 2s. 3d.; 50 yds. moreen at 3s. 3d.; 18 yds. dimity at 9½d.; 13 yds. diaper at 10½d.; 5 yds. satin ribbon at 1s. 1½d.; 17 yds. Swiss muslin at 11½d.

11. ½ bushel apples at 5s. 6d.; 50 oranges at 9s. 6d. per 100; 9 lbs. filberts at 1s. 7½d.; 3 doz. pears at 2s. 3d. per doz.; 20 lemons at 1½d.; a box of figs weighing 13 lbs. at 9½d.; and 3 pine apples at 5s. 3d.

12. 3 doz. linen shirts at 65s. per doz.; 2½ doz. socks at 9s.; 1½ doz. pocket handkerchiefs at 17s. 6d.; ½ doz. neck-ties at 27s.; 6 pair gloves at 3s. 9d.; 3 doz. collars at 9s. 6d.; and 1 railway rug at 14s. 9d.

13. 11 lbs. tea at 3s. 10d.; 8 lbs. coffee at 1s. 4d.; 32 lbs. sugar at 6½d.; a cheese weighing 37 lbs. at 8½d.; 3 hams weighing 25 lbs. each

at 9d.; 17 lbs. bacon at 10½d.; 22 lbs. butter at 1s. 3d.; 7 lbs. lard at 7d.; and 9 lbs. rice at 7½d.

14. September 4.—9 lbs. sugar at 5½d.; 1¼ lbs. tea at 4s. 6d.; 5 lbs. coffee at 1s. 9d.; and ¼ lb. mustard at 2s. 11d. September 30.—7 lbs. sugar at 6½d.; 5 lbs. loaf sugar at 9½d.; and 3 lbs. coffee at 1s. 10d. November 9.—4 lbs. tea at 4s. 8d.; 6 oz. pepper at 1½d.; 3 lbs. raisins at 6½d.; and 1½ lbs. currants at 9½d. Account paid, December 6.

15. A carpenter is employed by James Harris as follows:—May 4, 1857—Repairing garden paling, 13s. 6d.; 2 cucumber frames at £1. 5s. each; repairing flower stand, 8s. 6d. June 9.—2 deal tables at £1. 2s. 6d.; repairing window frame, 6s. 6d.; hanging pictures, 4s.; repairing skirting board, 9s. 6d. Aug. 6.—1 wicket gate for garden, 11s. 6d.; making and fixing trellis work, £2. 7s. 6d.

16. 11 yds. silk at 3s. 5d.; 6 pair gloves at 2s. 9d.; 3 pairs cotton stockings at 1s. 11d.; 12 yds. French merino at 3s. 10d.; 20 yds. long cloth at 8½d.; 15 yds. flannel at 2s. 5d.; 1 doz. cambric handkerchiefs at 1s. 9d. each; 9 yds. ribbon at 1s. 1½d.; 7 yds. lace at 2s. 9d.; and 27 yds. linen at 2s. 10d.

PRACTICE.

31. The half, third, fourth, &c., parts of any quantity are called *aliquot parts* of that quantity.

Practice is a short method of finding the value of any number of articles by means of aliquot parts. It is of two kinds, *Simple* and *Compound*. The questions which occur in Simple Practice are similar to those in Compound Multiplication.

In the following examples, Ex. 1 belongs to Simple Practice: Ex. 2 to Compound Practice.

Ex. 1. Find the value of 347 things at £2, 17s. 10d. each.

10s.	=½	347					
		2					
		694			=347 at	£2.	each.
5s.	=½	173	10		= „ at	10s.	„
2s. 6d.	=¼	86	15		= „ at	5s.	„
3d.	=$\frac{1}{10}$	43	7	6	= „ at	2s. 6d.	„
1d.	=⅓	4	6	9	= „ at	3d.	„
		1	8	11	= „ at	1d.	„
Ans.		£1003	8	2	= „ at	£2. 17s. 10d.	each.

Here we first multiply 347 by 2, which gives £694, or the value of

364 things at £2. each. We next take 10s. as $\frac{1}{2}$ of £1., and divide 347 by 2, which gives £173. 10s. Then 5s. as $\frac{1}{2}$ of 10s.; 2s. 6d. as $\frac{1}{2}$ of 5s., &c.; dividing the preceding line in each case.

The signs of equality on the right, as well as the statements to the right of those signs, are usually omitted in working questions. It is, however, advisable that questions should occasionally be fully worked as above.

Ex. 2. What is the rent of a farm consisting of 84 ac. 3 ro. 27 po., at £3. 3s. 4d. per acre.

2 ro. = $\frac{1}{4}$	£3	3	4	
			12 × 7 = 84	
	38	0	0	
			7	
	266	0	0	
1 ro. = $\frac{1}{2}$	1	11	8	
20 po. = $\frac{1}{4}$		15	10	
5 po. = $\frac{1}{4}$		7	11	
2 po. = $\frac{1}{10}$		1	11$\frac{1}{2}$	
			9$\frac{1}{2}$	
Ans.	£268	18	2$\frac{1}{2}$	

Here we find the value of 84 ac. by multiplication. We then take 2 ro. as $\frac{1}{4}$ of 1 ac., and divide £3. 3s. 4d. by 2. Next, 1 ro. = $\frac{1}{2}$ of 2 ro; 20 po. = $\frac{1}{4}$ of 1 ro., &c. It should be carefully noticed that 2 po. is taken as $\frac{1}{10}$ of 20 po., and therefore 7s. 11d. is divided by 10, *not* 1s. 11$\frac{1}{2}$d.

Ex. 3. Find the value of 229 articles at £2. 17s. 6d. each.

	229			
	3			
	687			= 229 at £3. each.
	28	12	6	= ,, at 2s. 6d. each.
	£658	7	6	= ,, at £2. 17s. 6d. each.

Since £2. 17s. 6d. is just 2s. 6d. less than £3., we here find the value of 229 at £3. each, and then of 229 at 2s. 6d. each, and *subtract* the latter value from the former. The same course may be taken with any question when the given price is any aliquot part of a pound less than an exact number of pounds. So also, when the price is given in shillings.

The aliquot parts of a pound are, 1s. 8d. = $\frac{1}{12}$; 2s. = $\frac{1}{10}$; 2s. 6d. = $\frac{1}{8}$; 3s. 4d. = $\frac{1}{6}$; 4s. = $\frac{1}{5}$; 5s. = $\frac{1}{4}$; 6s. 8d. = $\frac{1}{3}$; 10s. = $\frac{1}{2}$.

The pupil should be able to give the aliquot parts of a pound, shilling, ton, cwt., qr., &c., without hesitation. The ability to select the most suitable aliquot parts for the working of any question will be best obtained by practice.

Ex. 17.

Find the value of the following sets of articles at the assigned prices.

1. 29 at 4s. 6d.; 37 at 5s. 3d.; 68 at 6s. 8d.
2. 91 at 3s. 4d.; 45 at 2s. 6d.; 89 at 1s. 8d.
3. 53 at 2s. 3d.; 76 at 4s. 9d.; 97 at 7½d.
4. 26 at £1. 13s. 4d.; 49 at £3. 6s. 8d.; 40 at 17s. 6d.
5. 227 at £3. 8s. 2d.; 473 at £9. 11s. 6d.
6. 191 at £2. 5s. 11d.; 139 at £4. 15s. 1d.
7. 321 at £17. 13s. 4d.; 241 at £11. 17s. 6d.
8. 112 at £3. 16s. 8d.; 204 at £6. 14s. 9d.
9. 73 at £5. 2s. 9½d.; 302 at £1. 19s. 1½d.
10. 97 at £4. 18s. 3d.; 129 at £11. 16s. 1½d.
11. 248 at £9. 8s. 9½d.; 301 at £7. 12s. 10½d.
12. 963 at £1. 19s. 7½d.; 29 at £27. 17s. 4d.
13. 648 at £2. 10s. 11d. per doz.; 300 at 15s. 1d. per score.
14. 888 at £1. 19s. 2½d. per doz.; 460 at 19s. 1½d. per score.
15. 432 at £3. 4s. 9½d. per gross; 25 at £37. 9s. 10d. each.
16. 3 cwt. 3 qrs. 25 lbs. at £3. 10s. per cwt.
17. 5 tons 17 cwt. 3 qrs. 19 lbs. at £1. 10s. 4d. per cwt.
18. 27 ac. 3 ro. 31 po. at £3. 16s. 8d. per acre.
19. 34 lbs. 11 oz. 19 dwts. 21 grs. at £2. 10s. per oz.
20. 14 yds. 3 qrs. 3 na. at 9s. 4d. per yard.
21. 6 bus. 3 pks. 1 gal. 3 qts. at £2. 13s. 4d. per quarter.
22. 31 gals. of wine at £43. 6s. 3d. per hogshead.
23. 2 tons 13 cwt. 1 qr. 27 lbs. 15 oz. at £112. per ton.
24. Find the amount of wages due to 19 men for 17 days, at 3s. 10d. per day each.

Second Course.

1. A gentleman's income is £2. 15s. 10d. a-day; what is that a-year?
2. A grocer buys three chests of tea weighing 57 lbs. each at 2s. 9d. per lb.; 4 chests weighing 62 lbs. each at 2s. 10½d. per lb.; and 5 chests weighing 53 lbs. each at 3s. 5½d. per lb. What was the total cost?
3. What is the total value of 1112 bars of gold, each weighing 2 lbs. 10 oz. 13 dwt. at £3. 17s. 6d. per oz?
4. The rental of a parish is £71319: what is the total amount of the poor's rate at 8¾d. in the pound?
5. What is the worth of 3129 quills at 10s. 5d. per hundred?
6. Three pieces of cloth contain 63 yds., 57¼ yds., and 49¾ yds. respectively. The price of the first is 13s. 6d.; the second, 14s. 11d.; and the third, 15s. 9d. per yard. What does a clothier pay who buys the whole?
7. Half a million gallons of linseed oil at 6s. 3½d. per gal.?
8. A farm containing 341 acres is rented at £1. 16s. 8d. per acre;

the cost of cultivation is as follows:—57 acres of wheat cost £4. 10s. 8d. per acre; 69 acres of barley cost £2. 13s. 6d. per acre; 49 acres of oats cost £2. 16s. 10d. per acre; 25 acres of potatoes cost £7. 18s. per acre; 17 acres of turnips cost £4. 3s. 9d. per acre; the remainder is grass land which costs only 4s. 2d. per acre, and maintains 136 sheep, which were bought at £1. 15s. 6d. per head. The produce per acre from the arable land is as follows:—wheat, 5 qrs.; barley, 4 qrs. 4 bus.; oats, 6 qrs.; potatoes, 140 cwts.; turnips, 23 loads, one half of which are consumed by the sheep. The wheat is sold at 53s. per qr.; barley, at 36s. per qr.; oats, at 33s. per qr.; potatoes, at 3s. 4d. per cwt.; turnips at 13s. per load: and sheep at £2. 8s. per head. What is the annual profit?

REDUCTION TO UNITY.

32. The method of working questions similar to those usually given under Proportion, by the direct application of multiplication and division, has been called by French writers on Arithmetic *The Method of Reduction to Unity.*

Ex. 1. If 1 lb. of sugar costs $6\frac{1}{2}$d., what is the cost of 9 lbs.?

1 lb. costs $6\frac{1}{2}$d.

Multiply by 9, 9 lbs. cost $6\frac{1}{2}$d. $\times 9 = 4$s. $10\frac{1}{2}$d. *Ans.*

Ex. 2. If 11 lbs. of tea cost £2. 5s. 10d., what is that per lb.

11 lbs. cost £2. 5s. 10d.

Dividing by 11, 1 lb. costs £2. 5s. 10d. $\div 11 = 4$s. 2d. *Ans.*

Ex. 3. What must be given for two dozen oranges when 15 are bought for 1s. $10\frac{1}{2}$d.

15 cost 1s. $10\frac{1}{2}$d.

therefore 1 costs $1\frac{1}{2}$d.

therefore 24 cost $1\frac{1}{2}$d. $\times 24 = 3$s. *Ans.*

Instead of reducing 15 to 1 in this example we might proceed as follows—

15 cost 1s. $10\frac{1}{2}$d.

dividing by 5, 3 cost $4\frac{1}{2}$d.

multiplying by 8, 24 cost $4\frac{1}{2}$d. $\times 8 = 3$s. *Ans.*

It will be seen that 3 being a common factor of 15 and 24, it is sufficient to reduce the given number 15 to 3, instead of to unity. So also in all cases when the given number and proposed number have a common factor.

Ex. 4. If 4 men earn £28 in 6 weeks, in how many weeks will 9 men earn £42?

4 men earn £28 in 6 weeks.
therefore 1 man earns £7 in 6 wks.
therefore 1 man earns £42 in $6 \times 6 = 36$ wks.
therefore 9 men earn £42 in $36 \div 9 = 4$ wks. *Ans.*

It will be found convenient to arrange the numbers in each line, so that *the one which is like the required answer may stand last in the line.*

Instead of performing the multiplication or division at each step in the operation, it is often better to proceed as in the following example:—

Ex. 5. If 12 men mow 45 acres in 7 days of 10 hrs. each, how many acres will 14 men mow in 9 days of 8 hrs. each?

In 7 days of 10 hrs. 12 men mow 45 acres.

therefore, in 1 day of 10 hrs. 12 men mow $\frac{45}{7}$ acres.

therefore, in 9 days of 10 hrs. 12 men mow $\frac{45 \times 9}{7}$ acres.

therefore, in 9 days of 1 hr. 12 men mow $\frac{45 \times 9}{7 \times 10}$ acres.

therefore, in 9 days of 8 hrs. 12 men mow $\frac{45 \times 9 \times 8}{7 \times 10}$ acres.

therefore, in 9 days of 8 hrs. 1 man mows $\frac{45 \times 9 \times 8}{7 \times 10 \times 12}$ acres.

therefore, in 9 days of 8 hrs. 14 men mow $\frac{45 \times 9 \times 8 \times 14}{7 \times 10 \times 12}$ acres.

Which by cancelling, as shown on page 30, gives 54 acres as the answer.

When the pupil is able at once to perceive the relation of each term to the question, he may proceed thus:—

In 7 days of 10 hrs. 12 men mow 45 acres.

therefore, in 1 day of 1 hr. 1 man mows $\frac{45}{7 \,.\, 10 \,.\, 12}$ acres.

therefore, in 9 days of 8 hrs. 14 men mow $\frac{45 \,.\, 9 \,.\, 8 \,.\, 14}{7 \,.\, 10 \,.\, 12} = 54$ ac. *Ans.*

Ex. 18.

1. What is the price of 10 lbs. of coffee at the rate of 6s. 8d. for 4 lbs.?

2. The fare on a railway is half-a-crown for 20 miles; what should it be for 14 miles?

3. If 3 stone of sugar cost £1. 1s. 10½d., what should be given for 2½ cwt.?

4. If the 4lb. loaf is sold for 5d. when wheat is at 40s. per qr., what should be the price of the loaf when wheat is 56s. per qr.?

5. A profit of £1. 7s. 6d. is made on 12 yds. of cloth; what is the profit on 30 yds. at the same rate?

6. If 12 men can complete a piece of work in 10 days, in how many days can 15 men finish it?

7. If 12 men can complete a piece of work in 10 days, how many men will be required to perform twice as much in 5 days?

8. How many pounds of tea at 4s. 4d. per lb. should be given in exchange for 72 lbs. of sugar at 6½d. per lb.?

9. If 18 men can build a wall 42 yds. long in 5 days, what length of wall can be built by 35 men in 3 days?

10. If the 8d. loaf weighs 6 lbs. when wheat is 48s. per qr., what ought to be the price of wheat when the 6d. loaf weighs 4 lbs.?

11. If the interest on £450 for 14 yrs. is £92, what should be the interest on £350 for 9 yrs. at the same rate?

12. A bankrupt pays a dividend of 5s. 6d. in the pound, and a creditor thereby receives £99 in payment of a certain debt. Supposing the dividend had been half-a-crown more in the pound, what sum would the creditor have received if the debt had been twice as great?

13. If 8 men can mow 16 acres in 3 days, how many acres can 9 men mow in 5 days?

14. If 11 men earn £37. 2s. 6d. in 3 weeks, how much can they earn in 13 weeks?

15. If 12 men can finish a piece of work in 15 days, in how many days can 18 men finish it?

16. If 12 men can finish a piece of work in 15 days, working 9 hours a-day; how many days will 18 men require to finish it, working 15 hours a-day.

17. Supposing the revenue derived from the duty on tea is £4,000,000; what would be the increase in the revenue if, by reducing the duty one-half, the consumption is trebled?

18. A porter carries 2 cwt. 2 qrs. 7 lbs. a distance of 5 miles for 7s.; what weight ought he to carry the same distance for 11s.

19. The duty on an article is reduced from 10½d. to 7d., and the consumption is thereby doubled; what increase is there in the revenue, supposing it to have been £30,000 before the alteration?

20. A regiment of soldiers, consisting of 1,500 men, have provisions to last 20 days, at a daily allowance of 4 lbs. Supposing the regiment is increased by 500 recruits, what must be the daily allowance in order that the provisions may last 30 days?

21. The net rental of an estate, after deducting a property tax of 8d. in the pound, is £1,933. 6s. 8d., what is the gross rental?

22. A bankrupt pays 12s. 6d. in the pound; how much will a creditor lose on a debt of £800?

23. A bankrupt, whose assets amount to £1,593, pays 14s. 9d. in the pound; what is the amount of his debts?

24. A lent B £600 for 8 months; for what length of time should B lend A £800 in return, supposing the interest on money to be doubled?

33. Questions like the following, involving the application of the foregoing rules, are of frequent occurrence.

Ex. 1. A grocer mixes 10 lbs. of tea at 4s. per lb., with 6 lbs. at 5s. per lb. Find the price of a pound of the mixture.

10 lbs. at 4s. cost 40s.
6 lbs. at 5s. cost 30s.

therefore 16 lbs. cost 70s.

dividing by 16, we have 1 lb. of the mixture cost 4s. 4½d.

Ex. 2. How much water must be added to 12 gals. of brandy at 30s. per gal. to reduce the price to 24s. per gal.?

30s. × 12 = 360s. = price of whole.
360s. ÷ 24s. = 15 = no. of gals. of mixture.
15 — 12 = 3 gals. of water.

Ex. 3. Divide £43. 7s. 2½d. between A and B, so that A may have 4 times as much as B.

Here we have to divide the whole sum into 5 equal parts and give *four* to A and *one* to B.

£43. 7s. 2½d. ÷ 5 = £8. 13s. 5½d. = B's share.
£8. 13s. 5½d. × 4 = £34. 13s. 9d. = A's share.

After finding B's share, A's share might be found by subtracting B's from the whole sum. The result may be proved by adding the two shares together.

Suppose it is required to divide a sum of money into two parts, so that one may have as much again as the other. We should in this case divide the whole sum into *five* equal parts, and take *three* of those parts for the larger sum, and *two* parts for the lesser sum.

Suppose we have to divide a sum of money between A, B, and C,

so that A may have twice as much as C, and B twice as much as A. We should in this case have C's share = 1, A's = 1 × 2, B's = 2 × 3. Now 1 + 1 × 2 + 2 × 3 = 1 + 2 + 6 = 9. The sum therefore would be divided into 9 parts, *one* of which would be C's share; *two*, A's; and *six*, B's.

Ex. 19.

1. The quarterly income of a gentleman is £278. 16s. 9d., and he spends £648. 4s. 8¾d. a-year. How much does he save?

2. By what number must £49. 17s. 4¼d. be multiplied to make it £4438. 4s. 6¼d.?

3. Divide £3. 17s. 5¼d. into two parts, so that one may be twice as much as the other.

4. Powder is bought at 8s. 4d. per lb., and shot at 6d. per lb.: the charge of a gun is half an ounce of powder and two ounces of shot. If 7s. 6d. is spent in powder, how much should be spent in shot, so that both may be finished in the same number of shots?

5. Divide £63 into an equal number of guineas, half-guineas, sovereigns, half-sovereigns, crowns, half-crowns, and shillings.

6. A grocer buys 50 lbs. of tea at 4s. 6d. per lb., and 30 lbs. at 5s. 6d. per lb.; what is the average price per lb.?

7. If 7 workmen earn £400. 8s. in a year of 52 weeks, in how many weeks will two workmen earn £68. 4s.?

8. A piece of plate, composed of equal weights of gold and silver, costs £72. 3s. 9d.; what is the total weight, supposing the price of gold to be £3. 17s. 6d. per oz., and the price of silver to be 5s. per oz.?

9. What is the value of 288 articles at £2. 13s. 1d. each?

10. A grocer mixes 5 lbs. of tea at 3s. 2d. with 7 lbs. at 4s. 2d.: what should be the selling price per lb. so that he may gain 11s. on the whole?

11. If two men can earn as much as 5 boys, how much will 10 men and 11 boys earn in 9 days, the daily wage of a man being 3s. 9d.?

12. What sum of money must be added to £180. 11s. 2d., so that when the whole is multiplied by 5 the product may be just £42. 4s. 2d. less than £1000?

13. A gentleman's daily expenses are £4. 14s. 4d., and he saves £278. 8s. 4. a-year. What is his annual income?

14. Find, by Practice, the value of 3 tons 3 qrs. 24½ lbs. of sugar, at £2. 1s. 4d. per cwt.?

15. A gentleman's income, after deducting a property tax of 11d. in the pound, is £1431. 5s. What is his total income?

16. By increasing the duty on an article to 7d. per lb. the revenue is increased from £62100 to £86940. What was the increase per lb. in the duty?

17. If the price of wheat weighing 60 lbs. per bushel be 48s. per quarter, what should be given for a *cental* of corn when the price is half as much more?

18. It is required to mix brandy at 36s. per gallon with brandy at 30s. per gallon, so that a gallon of the mixture may be sold for 32s. 3d. How many pints of each kind will the gallon contain?

19. If 36 lbs. of bread serve 6 persons for 4 days, by how much must the daily allowance to each be reduced so that the same quantity may last 6 days?

20. The length of a railway is 99 miles: there is a station at a distance of 4½ miles from each terminus, and the same distance between all the intermediate stations. Two trains start at the same time, travelling at the rate of 22 and 18 miles an hour respectively; the fast train stopping 1 minute at each alternate station, and the slow train stopping 3 minutes at every station. How much sooner will the one perform the journey than the other?

21. A merchant sells one-third of a cargo of sugar at 6d. per lb., and afterwards, when the market has fallen 1½d. per lb., he sells the remainder, realizing £5600 more by the second sale than by the first. What was the weight of the cargo?

22. A merchant sells tin as follows:—2 tons of common block at £129 per ton; 4 tons of common bar at £130 per ton; and 6 tons of refined block at £137 per ton. What was the average price per ton on the whole?

23. Oil of vitriol is composed of 9 parts by weight of water, and 40 parts of dry sulphuric acid; dry sulphuric acid is composed of 16 parts by weight of sulphur, and 24 parts of oxygen. What weight of sulphur is there in a quantity of oil of vitriol which contains 49 lbs. 8 oz. of water?

24. A coal merchant buys 630 tons of coal, and by selling one-half clears a profit of £39. 7s. 6d.; but the market falling, he sells the remainder so as just to clear 15 guineas on the whole. What was the fall in the selling price per ton?

TABLES.

MONEY.

things...*f.* ...	make 1 penny	...*d.*
ace.....	„ 1 shilling	...*s.*
lings.....	„ 1 pound	...*£*

Proposed Decimal Coinage.

le...*m.*	make 1 cent.....	*c.*
its	„ 1 florin ...	*fl.*
rins	„ 1 pound.	

RRENT COINS. *Copper*—farthing; nny; penny. *Silver*—threepenny-; fourpenny-piece; sixpence; shil-florin=2s.; half-a-crown=2s. 6d.; =5s. *Gold*—sovereign; half-ign. The guinea is not now in but 21s. are often spoken of as a , and 10s. 6d. as half-a-guinea.

WEIGHT.

AVOIRDUPOIS WEIGHT.

ams, *drs.* make 1 ounce		*oz.*
nces ... „ 1 pound		*lb.*
unds ... „ 1 quarter.....		*qr.*
uarters... „ 1 hundredweight		*cwt.*
undredweight 1 ton.		

0, 14 lbs.=1 *stone*; 8 lbs.=1 stone itcher's meat in London; 100 lbs. *ental* of corn or flour in Liverpool ome other markets.
oirdupois weight is used for all ose goods.

TROY WEIGHT.

ains ...*gr.* make 1 pennywt....		*dwt.*
nts. „ 1 ounce.		
nces „ 1 pound.		

oy weight is used in weighing Gold Silver, and in philosophical ex-ents.

APOTHECARIES' WEIGHT.

20 grains.....	make 1 scruple...	*scr.*
3 scruples.....	„ 1 dram.	
8 drams.....	„ 1 ounce.	
12 ounces	„ 1 pound.	

This weight is used for medicines.
In Troy and Apothecaries' weight the *grain*, *ounce*, and *pound* are the same.
For the purpose of reduction it is useful to remember the following relations:—

1 lb. Avoirdupois=7000 grains.
1 lb. Troy =5760 grains.

LENGTH.

LINEAL MEASURE.

12 lines...*l*.....	make 1 inch.....	*in.*
12 inches	„ 1 foot	*ft.*
3 feet.....	„ 1 yard	*yd.*
5½ yards	„ 1 pole.....	*po.*
40 poles	„ 1 furlong ...	*fur.*
8 furlongs.....	„ 1 mile.....	*ms.*
3 miles	„ 1 league.	
69⅛ miles.....	„ 1 degree.	
60 geographical *m.*	„ 1 degree.	

Also, a *Hand*, used in measuring the height of horses=4 inches; a *Fathom*, used in measuring depth=6 feet.
The *chain* used in measuring land is 66 feet long, and divided into 100 *links*, a link being nearly 8 inches.

CLOTH MEASURE.

2¼ inches	make 1 nail.	
4 nails.....	„ 1 quarter ...	*qr.*
4 quarters	„ 1 yard	*yd.*
5 quarters	„ 1 English ell.	
6 quarters	„ 1 French ell.	
3 quarters	„ 1 Flemish ell.	

SURFACE.

Square Measure.

144 sq. in.	make	1 sq. foot.
9 sq. ft.	"	1 sq. yd.
30¼ sq. yds.	"	1 sq. pole.
40 poles	"	1 rood.
4 roods	"	1 acre.
640 acres	"	1 sq. mile.

Also,

100 sq. ft.	make	1 sq. of flooring.
272¼ sq. ft.	"	1 rod of brickwork.
36 sq. yds.	"	1 rod of building.
10 sq. chains	"	1 acre.
100000 sq. links	"	1 acre.

SOLIDITY.

Cubic Measure.

1728 cub. in.	make	1 cub. ft.
27 cub. ft.	"	1 cub. yd.

Also, 40 cub. ft.=1 ton of measurement in shipping. In measuring timber, 40 cub. ft. of rough, or 50 cub. ft. of hewn timber=1 load.

CAPACITY.

Corn or Dry Measure.

2 quarts	make	1 pottle.
2 pottles	"	1 gallon.
2 gallons	"	1 peck.
4 pecks	"	1 bushel.
8 bushels	"	1 quarter.
5 quarters	"	1 load.

It has recently been proposed to sell corn by weight. The name *cental* has been given to 100 lbs. weight, and is in use in Liverpool and other markets.

Ale and Beer Measure.

4 gills	make	1 pint.
2 pints	"	1 quart.
4 quarts	"	1 gallon.
9 gallons	"	1 firkin.
18 gallons	"	1 kilderkin
36 gallons	"	1 barrel.
54 gallons	"	1 hogshead

Wine Measure.

2 pints	make	1 quart.
4 quarts	"	1 gallon.
18 gallons	"	1 runlet.
42 gallons	"	1 tierce.
2 tierces	"	1 puncheon
63 gallons	"	1 hogshead,
2 hogsheads	"	1 pipe.
2 pipes	"	1 tun.

TIME.

60 seconds	make	1 minute.
60 minutes	"	1 hour.
24 hours	"	1 day.
7 days	"	1 week.
365 days	"	1 common year
366 days	"	1 leap year.

The true length of the year is 365 d 6 hrs. 48 m. 47 sec., nearly.

NUMBER.

12 units	make	1 dozen.
20 units	"	1 score.
12 dozen	"	1 gross.
24 sheets of paper	"	1 quire.
20 quires	"	1 ream.
10 reams	"	1 bale.

Angular Measure.

60 seconds	make	1 minute.
60 min.	"	1 degree.
90 deg.	"	1 right angle.

5 deg., 9 min., 17 sec., is written $5^{\circ}\ 9'\ 17''$.

PART II.

GREATEST COMMON MEASURE.

34. A *measure* of a number is a number which will divide the given number exactly. A *common measure* of two or more numbers is a number which will divide each of the given numbers exactly. The *greatest common measure* of two or more numbers is the greatest number which will divide each of the given numbers exactly.

Thus, 3 is a measure of 12, because it divides 12 without remainder. Also 3 is a common measure of 12 and 18, because it divides both 12 and 18 without remainder. But 6 is the greatest common member of 12 and 18, because it is the greatest number which will divide both 12 and 18 without remainder.

The letters G.C.M. are used for Greatest Common Measure.

To find the G.C.M. of any two numbers.

RULE. Divide the greater number by the less, the less by the remainder, the first remainder by the second, the second by the third, and so on till the division terminates: the last remainder or divisor is the G.C.M. required.

Find the G.C.M. of 224, and 316.

```
224)316(1
    224
    ---
     92)224(2
        184
        ---
         40)92(2
            80
            --
            12)40(3
               36
               --
                4)12(3
                  12
                  --
Ans. 4.
```

Or thus,

```
224(2
184
---
 40(3
 36
 --
Ans. 4
```

```
316(1
224
---
 92(2
 80
 --
 12(3
 12
 --
```

The greater number 316 is the first dividend, and the lesser one 224 is the divisor; the lesser number 224 is the next dividend, and the first remainder 92 is the divisor; the first remainder 92 is the next dividend, and the second remainder 40 is the divisor, &c.

The operation is the same in the working on the right hand, the dividend and divisor being alternately in the right and left columns.

To find the G.C.M. of three numbers.—Find the G.C.M. of any two, and then find the G.C.M. of the first G.C.M. and the remaining number.

To find the G.C.M. of four numbers.—Find the G.C.M. of any two, then of the remaining two, and, finally, find the G.C.M of the two first greatest common measures.

Similarly, for five or more numbers.

The chief use of this rule in arithmetic is in the reduction of fractions to their lowest terms.

Ex. 20.

Find the G.C.M. of

1.	24 and 42.	2.	39 and 65.	3.	68 and 102.
4.	57 and 209.	5.	63 and 47.	6.	52 and 143.
7.	187 and 493.	8.	1358 and 97.	9.	261 and 638.
10.	1113 and 1961.	11.	2035 and 2479.	12.	2619 and 7663.
13.	4641 and 5940.	14.	33649 and 29172.	15.	57178 and 29317.

16. 456483 and 1853603 17. 319074 and 520137.

18. 201, 737, and 871. 19. 648, 840, 897, and 663.

20. A gentleman wishes to distribute £12 among some poor men and 10 guineas among an equal number of poor women. What is the greatest number of each he can thus relieve by giving to each one an exact number of shillings?

21. The fore wheel of a coach makes the same number of revolutions in 156 feet as the hind wheel makes in 221 feet. Find the circumferences of the wheels, each being an exact number of feet.

22. A's wages amount to £7 5s. in the same time that B's amount to £5 16s. What are their daily wages, each being so many shillings, and the sum of their daily earnings less than 10s?

LEAST COMMON MULTIPLE.

35. A *multiple* of a number is a number which will contain the given number an exact number of times. A *common multiple* of two or more numbers is a number

which contains each of the given numbers exactly. The *least common multiple* of two or more numbers is the least number which contains each of the given numbers exactly.

Thus, 24 is a *multiple* of 8, because it contains 8 exactly three times. Again, 36 is a *common multiple* of 6 and 9, because it contains 6 six times and 9 four times without remainder. But 18 is the *least common multiple* of 6 and 9, because it is the *least* number which contains both 6 and 9 exactly.

The letters L.C.M. are used for Least Common Multiple.

To find the L.C.M. of two or more numbers.

RULE. (1.) Arrange the numbers in a line, striking out those that are contained in any of the others. (2.) Divide the remaining numbers by any number which will exactly divide more than one of them, placing the quotients under their respective numbers. (3.) Any number not exactly divisible by the divisor, yet containing a factor common to the divisor, must be divided by that common factor, and the quotient placed under it. (4.) All numbers prime to the divisor must be brought down unaltered. The second line will now be completed, and if any of the numbers forming it are contained in any of the others, they may be struck out as in the first line, the remaining numbers being divided as in the first line. Proceed in this way till all the remaining numbers are prime to each other. The continued product of all the divisors, and the remaining prime numbers is the L.C.M. required.

Ex. 1. Find the L.C.M. of 6, 16, 12, 15, 24, 8, and 30.
Ex. 2. Find the L.C.M. of 16, 24, 18, 42, 56, and 54.

Ex. (1.) 8)~~6~~, 16, ~~12~~, ~~15~~, 24, ~~8~~, 30
2, 3, 15

L.C.M. = 8 × 2 × 15 = 240 *Ans.*

Ex. (2.) 8)16, 24, ~~18~~, 42, 56, 54
3) 2, 3, 21, 7, 27
2, 7, 9

L.C.M. = 8 × 3 × 2 × 7 × 9 = 3024 *Ans.*

In Ex. 1. we have to find a number which will contain 6, 16, 12, 15, 24, 8, and 30; or, in other words, a number which can be divided by any one of these without remainder. Now any number which contains 24, will also contain 6, 8, and 12; and any number which contains 30, will contain 15; hence 6, 8, 12, 15, are struck out. Again, any number which contains the factors 8 and 2, will contain 16; and any number which contains the factors 8 and 3, will contain 24; but 15 contains 3, therefore any number containing 8 and 15, will contain 24; finally, any number containing the factors 2 and 15, will contain 30; but 2 and 15 are among the remaining numbers. Hence all the factors in the required multiple are 8, 2, and 15.

The division of 30 by the common factor 2 in Ex. 1., and of 42 and 54 by the common factor 2 in Ex. 2., should be carefully noticed. The use of the smallest possible divisors avoids this, but it is tedious.

Ex. 3. What is the least sum of money which can be made up exactly of crowns, of half-guineas, or of seven-shilling pieces?

1 cr. = 10 sixpences, 1 hlf.-gui. = 21 sixpences, 7s. = 14 sixpences. The L.C.M. of 10, 21, 14, is 210, and 210 sixpences = £5 5s.—*Ans.*

This rule is chiefly used in the reduction of fractions to their least common denominator.

Ex. 21.

Find the L.C.M. of—

1. 18, 12, 24.
2. 15, 20, 12.
3. 6, 9, 12, 24.
4. 8, 14, 24, 21.
5. 21, 33, 42, 14.
6. 9, 12, 27, 30.
7. 16, 48, 36, 32.
8. 38, 57, 36, 76.
9. 87, 69, 116, 92.
10. 39, 26, 78, 13.
11. 18, 24, 36, 48, 144.
12. 16, 28, 36, 35, 45.
13. 8, 18, 27, 12, 15, 40.
14. 2, 3, 4, 5, 6, 7, 8, 9, 10, 12.
15. 21, 25, 27, 28, 35, 36, 45.
16. 33, 84, 77, 28, 56, 48.
17. 15, 18, 45, 60, 90, 180.
18. 36, 70, 162, 110, 144, 84.
19. 34, 51, 39, 52, 38, 57, 102.
20. 15, 46, 40, 28, 115, 69, 112.
21. 143, 105, 187, 153, 364, 396, 765, 665.
22. 391, 779, 697, 437, 943, 481, 1517, 629.

23. What is the least sum of money which can exactly be made up of florins, of half-crowns, of guineas, or of moidores?

24. A's wages are 3s. per day; B's 4s. 6d.; C's 5s. In how many weeks of 6 days, will each have earned an exact number of pounds?

25. What is the capacity of the smallest sized cask that can be exactly filled by a 4 gallon, a 6 gallon, or a 9 gallon measure?

26. A government train travelling 20 miles, a mail train 30 miles, and an express train 36 miles per hour, all performing the distance between London and York in an exact number of hours: what is the least possible distance between the two places?

VULGAR FRACTIONS.

36. A *fraction* is a part of unity or of any integer. It is formed by two numbers, one placed over the other, with a line drawn between them, thus:—$\frac{3}{10}$, *three-tenths*

The lower number is called the *Denominator*, and shows into how many parts the unit is divided: the upper number is called the *Numerator*, and shows how many of those parts are taken.

Or we may regard the numerator as a whole number, and the denominator as showing into how many parts that number is divided, the fraction itself representing one such part.

Thus, $\frac{3}{4}$ may be regarded as three-fourths of 1, or as one-fourth of 3.

For three-fourths of 1s. = 9d., and one-fourth of 3s. also = 9d.

Hence, the numerator and denominator are related as dividend and divisor.

A *Proper fraction* has its numerator less than its denominator.

An *Improper fraction* has its numerator greater than its denominator.

Such forms as $\frac{3}{3}$, $\frac{12}{4}$, are, strictly speaking, not fractions, being only fractional in *form* and not in *value*.

A proper fraction might also be defined as one which is less, and an improper fraction as one which is greater than unity.

A *Simple fraction* has a whole number for each term.

A *Mixed number* consists of a whole number and a fraction.

A *Complex fraction* has a fraction or a mixed number in either or in both its terms.

A *Compound fraction* is a fraction of a fraction.

Examples. A proper fraction, $\frac{7}{8}$. An improper fraction, $\frac{12}{4}$. A mixed number, $4\frac{3}{5}$. Complex fractions, $\frac{\frac{1}{2}}{4}$, $\frac{3\frac{1}{2}}{\frac{3}{4}}$, $\frac{3\frac{1}{2}}{7\frac{1}{4}}$. A compound fraction, $\frac{1}{2}$ of $\frac{1}{3}$ of $2\frac{1}{4}$. Simple fractions, $\frac{1}{2}$, $\frac{3}{7}$, $\frac{12}{5}$.

Vulgar Fractions are so called because they may have any numbers for denominators, in contradistinction to *Decimal Fractions*, whose denominators are restricted to 10 and its powers.

Fractions containing large numbers are usually read thus, $\frac{307}{401}$, *three hundred and seven over four hundred and one.* Similarly with complex fractions, $\frac{3\frac{1}{7}}{\frac{2}{3}}$ *three and one-seventh over two-thirds.*

The sign $\therefore$ is used for the word *therefore.*

37. Since the numerator and denominator of a fraction are related as dividend and divisor, that is $\frac{3}{4}=3\div4$, it follows,—

(1.) That unity may be expressed in a fractional form with any number for both numerator and denominator.

For $\frac{7}{7}=7\div7=1$, $\therefore 1=\frac{7}{7}$; similarly, $1=\frac{4}{4}=\frac{13}{13}$, &c.

(2.) Any whole number may be put in a fractional form with unity for its denominator.

For $6=6\div1=\frac{6}{1}$; similarly, $9=\frac{9}{1}$, $15=\frac{15}{1}$, &c.

38. *To multiply a fraction by a whole number,*—(1.) Multiply the numerator of the fraction by the whole number; or, (2.) divide the denominator by it.

Ex. 1. $\frac{3}{4}\times3=\frac{9}{4}$. Ex. 2. $\frac{5}{4\,8}\times6=\frac{5}{8}$.

For in $\frac{3}{4}$ and $\frac{9}{4}$ the unit is divided into the same number of parts, but three times as many parts are taken in $\frac{9}{4}$ as in $\frac{3}{4}$. Again, the same number of parts are taken in $\frac{5}{4\,8}$ and $\frac{5}{8}$; but one of the latter parts is six times as great as one of the former. The first form of the rule is applied in Ex. 1., and the second form in Ex. 2. The second form should always be applied when practicable.

To divide a fraction by any whole number,—(1.) Divide the numerator by the whole number; or, (2.) multiply the denominator.

We have just seen that $\frac{3}{4}\times3=\frac{9}{4}$, $\therefore \frac{9}{4}\div3=\frac{3}{4}$. Also $\frac{5}{4\,8}\times6=\frac{5}{8}$, $\therefore \frac{5}{8}\div6=\frac{5}{4\,8}$. The first form of the rule should always be applied when practicable.

39. *To express any whole number in a fractional form with a given denominator,*—Multiply the whole number by

the given denominator for the numerator of the fraction, and write the given denominator under the product.

Examples. Express 6, 7, and 4, in a fractional form, with denominators 5, 4, and 11, respectively—

(1.) $6 = \frac{6 \times 5}{5} = \frac{30}{5}$; (2.) $7 = \frac{28}{4}$; (3.) $4 = \frac{44}{11}$.

For $6 = 1 \times 6 = \frac{5}{5} \times 6 = \frac{30}{5}$.

Ex. 22.

1. Multiply $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{5}{9}$, and $\frac{3}{10}$, each by 3.
2. Multiply $\frac{11}{14}$, $\frac{13}{17}$, and $\frac{21}{35}$, by 7, 4, and 11, respectively.
3. Divide $\frac{1}{2}$, $\frac{3}{5}$, $\frac{5}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$, each by 2.
4. Divide $\frac{9}{10}$, $\frac{13}{17}$, and $\frac{32}{40}$, by 3, 5, and 13, respectively.
5. Express 7, 8, 9, 10, and 17, as fractions, with denominators 9.
6. Express 13, 15, and 23, as fractions, with denominators 23, 12, and 13, respectively.
7. Express 9, 14, and 24, as fractions, with denominators 16, 15, and 28, respectively, and multiply each fraction by 4.
8. Multiply $\frac{13}{16}$, $\frac{11}{12}$, $\frac{15}{33}$, and $\frac{33}{35}$, each by 8, and divide each result by 11.

40. *To reduce an improper fraction to a whole or mixed number.*

Rule. Divide the numerator by the denominator, and write the quotient as a whole number, placing the remainder (if any) as the numerator of the fractional part, and the original denominator underneath it.

Exs. Reduce $\frac{11}{4}$, $\frac{29}{7}$, and $\frac{29}{9}$, to whole or mixed numbers.

$\frac{11}{4} = 2\frac{3}{4}$ *Ans.* $\frac{29}{7} = 4$ *Ans.* $\frac{29}{9} = 3\frac{2}{9}$ *Ans.*

For $\frac{11}{4} = \frac{8}{4}$ and $\frac{3}{4}$, and $\frac{8}{4} = 2$, therefore $\frac{11}{4} = 2\frac{3}{4}$.

41. *To express a mixed number as an improper fraction.*

Rule. Multiply the whole number by the denominator of the fractional part, and add the numerator of the fractional part to the product; this result is the numerator of the improper fraction: the denominator of the fractional part is retained as denominator.

Exs. Express $5\frac{3}{4}$ and $2\frac{3}{5}$ as improper fractions.

$$5\tfrac{3}{4} = \frac{5\times 4+3}{4} = \frac{20+3}{4} = \frac{23}{4}\ \textit{Ans.} \qquad 2\tfrac{3}{5} = \frac{13}{5}\ \textit{Ans.}$$

For $5 = \frac{20}{4}$, (by 39), therefore $5\frac{3}{4} = \frac{20}{4}$ and $\frac{3}{4}$, or $\frac{23}{4}$.

Ex. 23.

1. Reduce $\frac{13}{4}$, $\frac{11}{8}$, $\frac{19}{8}$, $\frac{29}{6}$, $\frac{35}{7}$, $\frac{43}{8}$, $\frac{97}{11}$, to whole or mixed numbers.
2. Express $3\frac{5}{8}$, $4\frac{3}{7}$, $11\frac{1}{3}$, and $13\frac{1}{5}$, as improper fractions.
3. Reduce $\frac{29}{17}$, $\frac{97}{13}$, $\frac{1253}{40}$, $\frac{3117}{10}$, and $\frac{1000}{215}$ to mixed numbers.
4. Express $13\frac{5}{8}$, $23\frac{5}{7}$, $59\frac{4}{11}$, and $75\frac{3}{5}$, as improper fractions.
5. Multiply $\frac{13}{15}$, $\frac{15}{16}$, $\frac{19}{40}$, $\frac{13}{17}$, $\frac{23}{56}$, $\frac{11}{13}$, $\frac{71}{96}$, and $\frac{19}{20}$, each by 8, and express the results as mixed numbers.
6. Express $1\frac{4}{5}$, $2\frac{3}{4}$, $4\frac{5}{8}$, and $2\frac{3}{17}$, as improper fractions, and divide each result by 3.
7. Divide $\frac{34}{5}$, $\frac{45}{7}$, $\frac{70}{11}$, $\frac{96}{13}$, and $\frac{1325}{19}$, each by 5, and reduce the results to mixed numbers.
8. Find a fraction whose numerator is the G.C.M. of 323 and 471, and its denominator the L.C.M. of 4, 6, 9, 8, 12, and 16, and multiply the fraction by 72.

42. *If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the value of the fraction remains the same.*

Ex. $\frac{3}{4} = \frac{3\times 2}{4\times 2} = \frac{6}{8}$ for in $\frac{3}{4}$ the unit is divided into 4 parts, while in $\frac{6}{8}$ it is divided into 8 parts; therefore one of the former parts must be equal to two of the latter, and consequently, 3 of the former must $=6$ of the latter; that is, $\frac{3}{4} = \frac{6}{8}$.

Conversely $\frac{6}{8} = \frac{3}{4}$; that is, dividing the numerator and denominator of $\frac{6}{8}$ each by 2, the value of the fraction remains the same.

43. A fraction is said to be in its *lowest terms* when its numerator and denominator are prime to each other, that is, contain no common factor greater than unity.

To reduce a fraction to its lowest terms.

RULE. Divide both numerator and denominator by their G.C.M.

Ex. 1. Reduce $\frac{85}{153}$ to its lowest terms.

The G.C.M. of 85 and 153 is 17, therefore dividing both by this we have $\frac{85}{153}=\frac{5}{9}$ *Ans.*

A fraction may frequently be reduced to its lowest terms by *inspection*, either at once, or by successive divisions.

Ex. 2. Reduce $\frac{24}{36}$ and $\frac{4338}{10008}$ to their lowest terms.

It is evident that 12 is the G.C.M. of 24 and 36, therefore, $\frac{24}{36}=\frac{2}{3}$ *Ans.*

Again, $\frac{4338}{10008}=\frac{1084}{2502}$, dividing by 4.
$=\frac{542}{1251}$, dividing by 4.
$=\frac{62}{139}$, *Ans.* dividing by 9.

It is useful to remember that no fraction can be in its lowest terms whose numerator and denominator are both *even* numbers, since all even numbers are divisible by 2. Neither can any fraction be in its lowest terms whose numerator and denominator both end in 0, both in 5, or one in 0 and the other in 5, all such numbers being divisible by 5.

44. *To reduce a compound fraction to a simple one.*

RULE. Multiply all the numerators together for the numerator and all the denominators together for the denominator of the simple fraction.

Mixed numbers must be reduced to improper fractions before the rule can be applied. It is also convenient to represent whole numbers as fractions with denominator unity. The resulting simple fraction must be given in its lowest terms, and if it be an improper fraction it must be reduced to a whole or mixed number.

Ex. 1. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{5}$, and $1\frac{1}{2}$ of $\frac{2}{7}$ of 6 to their simplest forms.

(1.) $\frac{1}{2}\times\frac{3}{4}\times\frac{4}{5}=\frac{12}{40}=\frac{3}{10}$ in its lowest terms. *Ans.*
(2.) $1\frac{1}{2}$ of $\frac{2}{7}$ of $6=\frac{3}{2}\times\frac{2}{7}\times\frac{6}{1}=\frac{36}{14}=3$. *Ans.*

For the half of $\frac{3}{4}=\frac{3}{4}\div 2=\frac{3}{8}$, and $\frac{1}{5}$ of $\frac{1}{2}=\frac{1}{2}\div 8=\frac{1}{40}$.
$\therefore\ \frac{3}{8}$ of $\frac{1}{2}=\frac{12}{40}=\frac{3}{10}$, that is $\frac{1}{5}$ of $\frac{3}{4}$ of $\frac{4}{5}=\frac{3}{10}$.

This process may be shortened by employing the method of *cancelling* explained in (20), and striking out any factor common to a numerator and denominator.

Ex. 2. Reduce $\frac{4}{7}$ of $\frac{5}{8}$ of $1\frac{8}{11}$ of $4\frac{2}{5}$ of $\frac{1}{2}$ to its simplest form.

$$\frac{\cancel{4}^{1}}{\cancel{7}_{1}}\times\frac{\cancel{5}^{1}}{\cancel{8}_{\cancel{2}}}\times\frac{\cancel{19}^{\cancel{7}}}{\cancel{11}_{1}}\times\frac{\cancel{22}^{2}}{\cancel{5}_{1}}\times\frac{1}{\cancel{2}_{1}}=1\ \textit{Ans.}$$

Ex. 24.

1. Reduce $\frac{12}{18}$, $\frac{28}{42}$, $\frac{15}{35}$, $\frac{84}{90}$, and $\frac{111}{185}$, to their lowest terms.
2. Reduce $\frac{208}{160}$, $\frac{115}{391}$, $\frac{210}{415}$, and $\frac{875}{670}$, to their lowest terms.
3. Reduce $\frac{3}{4}$ of $\frac{5}{6}$, and $\frac{3}{7}$ of $\frac{5}{8}$ to simple fractions.
4. Reduce $\frac{3}{7}$ of $\frac{5}{8}$ of $\frac{14}{15}$ of $\frac{21}{25}$ to a simple fraction.

Reduce the following fractions to their simplest form:—

5. $\frac{3}{5}$ of $1\frac{2}{5}$ of $\frac{10}{11}$.
6. $1\frac{3}{4}$ of $3\frac{1}{7}$ of $\frac{4}{5}$ of 12.
7. $\frac{24}{35}$ of $\frac{21}{32}$ of $\frac{55}{56}$ of $\frac{48}{55}$.
8. $\frac{11}{12}$ of $3\frac{1}{3}$ of $\frac{4}{55}$ of 6.
9. $7\frac{1}{5}$ of $\frac{3}{10}$ of $4\frac{1}{7}$.
10. $3\frac{1}{3}$ of $2\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$.
11. $\frac{5}{65}$ of $5\frac{1}{4}$ of $\frac{4}{17}$ of $\frac{3}{5}$.
12. $\frac{39}{47}$ of $\frac{33}{65}$ of $6\frac{4}{15}$ of 135.
13. Reduce to their lowest terms $\frac{1845}{2184}$, $\frac{3339}{5661}$, $\frac{2076}{13013}$, and $\frac{10101}{14355}$.
14. Reduce $3\frac{1}{4}$ of $1\frac{3}{15}$ of $\frac{11}{13}$ of $\frac{14}{33}$ of $4\frac{3}{11}$ of $\frac{13}{119}$ to its simplest form.
15. How much is $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{2}{4}$ of a shilling?
16. Reduce $\frac{100455}{663663}$ to its lowest terms, and multiply it by 33.

45. *To reduce fractions to equivalent ones with their least common denominator.*

RULE. Find the L.C.M. of the denominators for a common denominator: divide this common denominator by the original denominators, and multiply the numerators by the respective quotients.

Ex. 1. Reduce $\frac{5}{6}$, $\frac{1}{9}$, $\frac{7}{12}$, to their least common denominator.

The L.C.M. of 6, 9, 12, is 36; dividing 36 by 6, 9, 12, we obtain the quotients 6, 4, 3; multiplying 5, 1, 7, by 6, 4, 3, respectively, we have 30, 4, 21, as the new numerators; consequently, the required fractions are, $\frac{30}{36}$, $\frac{4}{36}$, $\frac{21}{36}$, more conveniently written $\frac{30,\ 4,\ 21}{36}$, this latter form being read,—*thirty, four, and twenty-one, over thirty-six.*

This operation is merely multiplying the numerator of each fraction by the number obtained by dividing the common denominator by its own denominator.

Ex. 2. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{10}$, $\frac{8}{15}$, $\frac{11}{12}$, to their least common denominator.

L.C.M. = 120, $\therefore \frac{3}{4}, \frac{5}{6}, \frac{7}{10}, \frac{8}{15}, \frac{11}{12} = \frac{90,\ 100,\ 105,\ 96,\ 110}{120}$ *Ans.*

Compound fractions must be reduced to simple ones before the rule can be applied.

Ex. 3. Reduce $\frac{1}{7}$ of $\frac{4}{5}$ and $1\frac{2}{15}$ to their least common denominator.

$\frac{1}{7}$ of $\frac{4}{5} = \frac{4}{35}$; $1\frac{2}{15} = \frac{28}{70}$; *Ans.*, $\frac{28}{70}, \frac{28}{70}$.

By means of this rule the relative values of fractions may be compared.

For if the fractions be reduced to a common denominator, then the values of the fractions are as their numerators.

Ex. 4. Arrange the fractions $1\frac{3}{16}$, $3\frac{3}{4}$ of $\frac{3}{11}$, and $\frac{17}{24}$, in the order of their value, beginning with the greatest.

$1\frac{3}{16}$ $= \frac{19}{16}$.

$3\frac{3}{4}$ of $\frac{3}{11} = \frac{15}{4}$ of $\frac{3}{11} = \frac{9}{4}$ $= \frac{18}{16}$.

$\frac{17}{24}$ $= \frac{11}{16}$.

∴ the fractions stand $1\frac{3}{16}$, $\frac{17}{24}$, $3\frac{3}{4}$ of $\frac{3}{11}$ *Ans.*

Ex. 25.

Reduce to their least common denominator—

1. $\frac{3}{4}, \frac{5}{6}, \frac{1}{12}$.
2. $\frac{3}{4}, \frac{1}{2}, \frac{3}{5}$.
3. $\frac{3}{7}, \frac{5}{14}, \frac{1}{6}, \frac{2}{21}$.
4. $\frac{3}{5}, \frac{7}{10}, \frac{2}{15}, \frac{3}{25}$.
5. $\frac{5}{8}, \frac{2}{11}, \frac{1}{2}, \frac{3}{4}$.
6. $\frac{7}{9}, \frac{2}{14}, \frac{1}{18}, \frac{5}{21}$.
7. $\frac{2}{15}, \frac{3}{4}, \frac{4}{25}, \frac{4}{100}$.
8. $\frac{7}{12}, \frac{5}{12}, \frac{2}{27}, \frac{5}{36}$.
9. $\frac{2}{3}, \frac{1}{7}, \frac{1}{14}, \frac{1}{18}$.
10. $\frac{2}{3}, \frac{4}{5}, \frac{5}{7}, \frac{1}{15}$.
11. $\frac{7}{18}, \frac{9}{10}, \frac{5}{24}, \frac{13}{24}$.
12. $\frac{4}{27}, \frac{11}{30}, \frac{17}{45}, \frac{13}{45}$.
13. $\frac{4}{15}, \frac{17}{60}, \frac{23}{30}, \frac{29}{45}, \frac{31}{60}$
14. $\frac{11}{13}, \frac{17}{65}, \frac{15}{52}, \frac{14}{117}, \frac{5}{39}, \frac{29}{130}$.
15. $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{12}$.
16. $\frac{3}{5}$ of $\frac{4}{9}$, and $\frac{3}{7}$ of $\frac{4}{15}$.
17. $\frac{4}{21}, \frac{13}{56}, \frac{7}{24}, \frac{41}{119}, \frac{53}{152}, \frac{5}{68}$.
18. $\frac{3}{7}, \frac{3}{5}, \frac{5}{11}, \frac{10}{19}, \frac{7}{13}, \frac{3}{5}$.

Arrange according to their value—

19. $\frac{3}{8}$ of $1\frac{1}{3}$, $\frac{11}{13}$, $\frac{17}{21}$, $\frac{27}{45}$.
20. $\frac{29}{45}$, $\frac{4}{5}$ of $1\frac{7}{17}$, $\frac{11}{15}$.

ADDITION OF FRACTIONS.

46. Rule. Reduce the fractions (if necessary) to their least common denominator, and add the new numerators together.

Ex. 1. Add together $\frac{3}{4}, \frac{5}{6}, \frac{1}{3}, \frac{4}{5}, \frac{7}{12}$.

$\frac{3}{4} + \frac{5}{6} + \frac{1}{3} + \frac{4}{5} + \frac{7}{12} = \frac{54+46+12+32+42}{72} = \frac{185}{72} = 2\frac{41}{72}$ *Ans.*

Ex. 2. $3\frac{1}{8}$ of $\frac{7}{8}$ of $\frac{1}{3}+\frac{9}{20}+\frac{5}{24}+\frac{7}{9}+\frac{3}{11}$ of $2\frac{1}{16}$.

$$=\frac{\overset{2}{16}}{5}\times\frac{7}{\cancel{8}}\times\frac{1}{3}+\frac{9}{20}+\frac{5}{24}+\frac{7}{9}+\frac{3}{11}\times\frac{\overset{3}{33}}{16}.$$

$$=\tfrac{14}{15}+\tfrac{9}{20}+\tfrac{5}{24}+\tfrac{7}{9}+\tfrac{9}{16}=\frac{672+324+150+560+405}{720}.$$

$$=\tfrac{2111}{720}=2\tfrac{671}{720}\ \textit{Ans.}$$

Where mixed numbers occur the integral parts may be added separately.

Ex. 3. Add together, $3\frac{1}{4}+7\frac{1}{8}+\frac{11}{18}+5\frac{1}{4}$ of $\frac{3}{7}+1\frac{7}{12}$.

Reducing the compound fraction, $5\frac{1}{4}$ of $\frac{3}{7}=\frac{21}{4}$ of $\frac{3}{7}=2\frac{1}{4}$.

Adding the whole numbers, $3+7+2+1=13$.

Next, $\frac{1}{4}+\frac{1}{8}+\frac{11}{18}+\frac{1}{4}+\frac{7}{12}=\frac{18+9+44+18+42}{72}=\frac{131}{72}=1\frac{59}{72}$.

Finally, $13+1\frac{59}{72}=14\frac{59}{72}$ *Ans.*

Ex. 4. $\frac{11}{24}+2\frac{13}{32}+5\frac{17}{40}+3\frac{1}{48}=10\frac{220+195+204+10}{480}$.

$$=10\tfrac{629}{480}=10+1\tfrac{149}{480}=11\tfrac{149}{480}\ \textit{Ans.}$$

Ex. 26.

Add together,

1. $\frac{3}{4}$ and $\frac{1}{8}$.
2. $\frac{1}{7}$ and $\frac{3}{14}$.
3. $\frac{3}{5}$ and $\frac{4}{9}$.
4. $\frac{7}{12}$ and $\frac{5}{12}$.
5. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
6. $\frac{3}{10}$, $\frac{2}{5}$, $\frac{5}{6}$.
7. $\frac{7}{9}$, $\frac{1}{12}$, $\frac{3}{8}$.
8. $2\frac{1}{4}$ and $3\frac{1}{2}$.
9. $1\frac{1}{4}$, $2\frac{1}{2}$, $3\frac{1}{3}$.
10. $\frac{11}{14}$, $\frac{19}{21}$, $\frac{5}{6}$, $\frac{3}{7}$.
11. $2\frac{1}{5}+\frac{7}{10}+3\frac{4}{15}$.
12. $\frac{17}{24}$, $\frac{13}{16}$, $7\frac{7}{30}$.
13. $5\frac{1}{3}$, $\frac{11}{21}$, $6\frac{1}{7}$.
14. $2\frac{1}{5}$, $\frac{3}{7}$, $4\frac{2}{3}$.
15. $\frac{1}{15}$, $\frac{11}{12}+1\frac{13}{18}$.
16. $\frac{7}{8}$, $\frac{5}{9}$, $\frac{1}{12}$, $\frac{11}{18}$, $\frac{27}{48}$.
17. $\frac{4}{15}$, $3\frac{1}{10}$, $\frac{3}{8}$, $5\frac{1}{8}$, $\frac{9}{20}$.
18. $\frac{1}{8}$, $\frac{5}{12}$, $\frac{5}{6}$ of $\frac{3}{10}$.
19. $\frac{3}{4}$ of $\frac{3}{4}$, $\frac{5}{8}$ of $\frac{3}{7}$ of $\frac{14}{15}$, $\frac{1}{4}$.
20. $\frac{1}{6}$, $2\frac{1}{3}$, $3\frac{1}{7}$, $1\frac{1}{2}$, $\frac{7}{9}$.
21. $5\frac{1}{3}$ of $\frac{7}{8}$, $6\frac{1}{4}$, $\frac{1}{3}$ of $\frac{3}{4}$.
22. $3\frac{1}{4}$ of $2\frac{1}{3}$ of $1\frac{1}{20}$, $5\frac{1}{2}$, $\frac{7}{8}$.
23. $2\frac{11}{13}$, $\frac{10}{65}$, $\frac{22}{65}$, $\frac{3}{78}$, $\frac{5}{52}$.
24. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$.
25. $\frac{7}{10}$, $\frac{11}{12}$, $\frac{14}{15}$, $2\frac{3}{16}$, $\frac{7}{18}$.
26. $16\frac{1}{2}$, $23\frac{5}{10}$, $8\frac{43}{57}$, $1\frac{37}{152}$.
27. $\frac{5}{6}$, $\frac{3}{5}$, $\frac{5}{8}$, $1\frac{1}{4}$, $\frac{2}{7}$, $1\frac{1}{18}$.
28. $9\frac{3}{7}$ of $1\frac{1}{18}$ of $\frac{9}{10}+1\frac{5}{16}+\frac{3}{5}$ of $1\frac{1}{12}+1\frac{5}{36}+\frac{31}{36}$.
29. $\frac{14}{19}$, $\frac{15}{17}$, $\frac{3}{23}$, $\frac{5}{13}$.
30. $\frac{7}{9}$, $\frac{1}{10}$, $\frac{3}{13}$, $\frac{4}{21}$, $\frac{3}{29}$.

SUBTRACTION.

47. RULE. Reduce the fractions (if necessary) to their least common denominator, and take the lesser of the new numerators from the greater.

Ex. 1. From $\frac{7}{11}$ take $\frac{4}{11}$.

$\frac{7}{11}-\frac{4}{11}=\frac{3}{11}$ *Ans.* Here the fractions having a common denominator no reduction is required.

Ex. 2. Take $\frac{3}{4}$ from $\frac{9}{8}$. $\frac{9}{8}-\frac{3}{4}=\frac{9-6}{12}=\frac{1}{12}$ *Ans.*

Here $\frac{3}{4}$ is converted into $\frac{6}{12}$, and $\frac{9}{8}$ into $\frac{18}{12}$, by (45).

Ex. 3. From $3\frac{1}{4}$ take $2\frac{1}{5}$. This may be worked in two ways.

(1.) By converting the mixed numbers to improper fractions.

$$3\tfrac{1}{4}-2\tfrac{1}{5}=\tfrac{13}{4}-\tfrac{11}{5}=\tfrac{65-44}{20}=\tfrac{21}{20}=1\tfrac{1}{20}\ \textit{Ans.}$$

(2.) By subtracting the integral and fractional parts separately.

$$3\tfrac{1}{4}-2\tfrac{1}{5}=1\tfrac{1}{4}-\tfrac{1}{5}=1\tfrac{5-4}{20}=1\tfrac{1}{20}\ \textit{Ans.}$$

Ex. 4. Take $2\frac{1}{4}$ from $5\frac{1}{6}$. This may be worked in three ways.

(1.) $5\frac{1}{6}-2\frac{1}{4}=\frac{31}{6}-\frac{9}{4}=\frac{62-27}{12}=\frac{35}{12}=2\frac{11}{12}$ *Ans.*

(2.) $5\frac{1}{6}-2\frac{1}{4}=3\frac{1}{6}-\frac{1}{4}=3\frac{2}{12}-\frac{3}{12}=3-\frac{1}{12}=2\frac{11}{12}$ *Ans.*

In the expression $3\frac{2}{12}-\frac{3}{12}$ we take two of the three-twelvths from $\frac{2}{12}$, and then take the remaining $\frac{1}{12}$ from 3.

(3.) $5\frac{1}{6}-2\frac{1}{4}=3\frac{1}{6}-\frac{1}{4}=3\frac{2}{12}-\frac{3}{12}=2\frac{14}{12}-\frac{3}{12}=2\frac{11}{12}$ *Ans.*

Here we resolve the mixed number $3\frac{2}{12}$ into $2\frac{12}{12}+\frac{2}{12}$ which $=2\frac{14}{12}$.

Compound fractions must be reduced to simple ones.

Ex. 5. $\frac{11}{16}-\frac{3}{4}$ of $\frac{1}{3}=\frac{11}{16}-\frac{3}{20}=\frac{44-9}{60}=\frac{35}{60}=\frac{7}{12}$ *Ans.*

Ex. 6. From $5\frac{3}{7}$ take $3\frac{1}{8}$ of $\frac{4}{7}$ of $\frac{3}{8}$.

$$5\tfrac{3}{7}-\frac{\overset{2}{\cancel{16}}}{\cancel{56}}\times\frac{\cancel{56}}{7}\times\frac{3}{\cancel{8}}=5\tfrac{3}{7}-\frac{6}{7}=5\frac{14-18}{21}=5-\frac{4}{21}=4\tfrac{17}{21}\ \textit{Ans.}$$

Ex. 7. From $\frac{11}{16}+\frac{3}{7}$ of $4\frac{2}{8}$ take $1\frac{1}{4}+\frac{5}{8}$.

$$\frac{11}{16}+\frac{\cancel{3}}{\cancel{7}}\times\frac{\overset{2}{\cancel{14}}}{\cancel{8}}-1\frac{2+5}{8}=2\tfrac{11}{16}-1\tfrac{7}{8}=1\frac{11-14}{16}=1-\frac{3}{16}=\frac{13}{16}\ \textit{Ans.}$$

Ex. 27.

1. $\frac{5}{6}-\frac{1}{6}$.
2. $\frac{1}{2}-\frac{1}{3}$.
3. $\frac{3}{5}-\frac{1}{4}$.
4. $\frac{7}{12}-\frac{3}{12}$.
5. $\frac{1}{5}-\frac{1}{6}$.
6. $\frac{3}{4}-\frac{5}{8}$.
7. $\frac{7}{12}-\frac{5}{9}$.
8. $1\frac{1}{2}-\frac{1}{12}$.
9. $\frac{7}{11}-\frac{4}{7}$.
10. $1\frac{1}{4}-\frac{13}{20}$.
11. $\frac{23}{24}-\frac{31}{36}$.
12. $\frac{17}{35}-\frac{16}{35}$.
13. $4\frac{3}{4}-2\frac{1}{5}$.
14. $11\frac{1}{2}-7\frac{3}{5}$.
15. $6-3\frac{5}{18}$.
16. $2\frac{5}{9}-\frac{7}{12}$.
17. $13\frac{11}{18}-12\frac{13}{16}$.
18. $5\frac{5}{14}-3\frac{9}{10}$.
19. $\frac{13}{24}-\frac{3}{8}$ of $\frac{1}{16}$.
20. $\frac{3}{10}$ of $\frac{5}{9}-\frac{1}{15}$.
21. $13\frac{7}{16}-\frac{5}{6}$ of $2\frac{2}{5}$.
22. $3\frac{1}{2}$ of $\frac{5}{6}-1\frac{1}{18}$.
23. $7\frac{3}{11}-2\frac{1}{7}$ of $\frac{4}{7}$.
24. $26\frac{3}{14}-1\frac{1}{7}$ of $\frac{5}{9}$.
25. $3\frac{1}{2}$ of $1\frac{1}{3}$ of $1\frac{1}{2}\frac{1}{5}-4\frac{3}{8}$.
26. $\frac{3}{11}$ of $4\frac{3}{5}-\frac{4}{5}$ of $\frac{3}{7}$ of $3\frac{1}{3}$.
27. $217\frac{14}{15}-\frac{7}{9}$ of $5\frac{1}{8}$ of $14\frac{1}{7}$.
28. $\frac{11}{13}$ of $\frac{5}{7}$ of $\frac{3}{5}-2\frac{1}{6}$ of $\frac{7}{25}$ of $\frac{3}{16}$.

29. From $2\frac{1}{3}+\frac{7}{9}+4\frac{1}{4}$ take $1\frac{1}{8}+\frac{7}{9}+2\frac{1}{12}+\frac{3}{8}$.

30. From $\frac{3}{5}+\frac{3}{5}+\frac{5}{5}+1\frac{1}{5}$ take $\frac{5}{10}+1\frac{3}{4}+\frac{4}{100}$.

31. Take $7\frac{1}{8}$ of $3\frac{5}{8}$ of $\frac{17}{16}$ from $11\frac{1}{2}$ of $2\frac{15}{69}$ of $\frac{53}{7}$.

32. What number must be added to $\frac{1}{3}$ to make it $\frac{5}{11}$ of $4\frac{1}{8}$?

Second Course.

1. One-third of a ship belongs to A, one-fourth to B, two-ninths to C, and the remainder to D. What is D's share?

2. Add together, $\frac{17}{35}$, $3\frac{1}{8}$ of $\frac{4}{7}$ of $4\frac{3}{5}$, $\frac{19}{45}$, and $\frac{11}{21}$ of $\frac{102}{143}$.

3. From the difference of $5\frac{4}{7}$ of $2\frac{3}{13}$ and $7\frac{1}{8}$ of $\frac{4}{19}$, take the sum of $\frac{7}{15}$, $\frac{14}{25}$, $\frac{47}{45}$, and $\frac{5}{11}$ of $1\frac{3}{4}$ of $5\frac{1}{3}$.

4. One-half the passengers in a train are third class, one-third are second class, and the remaining 31 are first class. How many are there altogether?

5. Two-thirds of a 12-gallon cask consists of wine, and the remainder of water; one-fourth of the mixture is drawn off and the cask filled up with water. How many gallons of each does the cask now contain?

6. What fraction must be taken from $1\frac{1}{24}$ to leave $\frac{19}{51}$?

7. What fraction must be added to $\frac{5}{7}$ of $2\frac{1}{15}$ of $\frac{1}{12}$ of $1\frac{3}{76}$ to make it $1\frac{9}{16}$?

8. A, B, and C shoot at a mark; A hits it three times out of four, B twice out of five, and C four times out of seven. If they fire in turn, how many times will the mark be hit in 420 shots?

9. By how much is the fraction $\frac{3}{5}$ increased by adding 3 to both its terms?

10. By how much is the fraction $\frac{3}{4}$ diminished by subtracting 1 from both its terms?

11. Which is greater, $3\frac{1}{4}$ of $5\frac{1}{7}$ of $\frac{13}{11}$, or $4\frac{1}{2}$ of $3\frac{9}{7}$ of $\frac{5}{36}$?

12. A spirit merchant buys brandy at 36s. per gallon: how much water must be added to 7 gallons of brandy, so that by retailing it at the same price he may clear a profit of 8s. per gallon?

MULTIPLICATION.

48. RULE. Multiply all the numerators together for a new numerator, and all the denominators together for

a new denominator, and (if necessary) reduce the resulting fraction to its simplest form.

Ex. 1. Multiply $\frac{4}{7}$ by $\frac{3}{5}$. $\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$ *Ans.*

Ex. 2. Multiply $\frac{5}{8}$ by $\frac{10}{25}$. $\frac{5}{8} \times \frac{10}{25} = \frac{50}{200} = \frac{1}{4}$ *Ans.*

If we strike out the factors common to the numerators and denominators, the result will be at once in its lowest terms.

Ex. 3. Multiply $\frac{7}{9}$ by $\frac{15}{28}$. $\frac{\cancel{7}^{1}}{\cancel{9}_{3}} \times \frac{\cancel{15}^{5}}{\cancel{28}_{4}} = \frac{5}{12}$ *Ans.*

Mixed numbers must be reduced to improper fractions before the rule can be applied.

Ex. 4. $3\frac{1}{3} \times \frac{5}{17} = \frac{10}{3} \times \frac{5}{17} = \frac{50}{51}$ *Ans.*

Ex. 5. Multiply $5\frac{1}{4}$ of $3\frac{2}{3}$ by $\frac{4}{7}$ of $\frac{2}{11}$.

$$5\tfrac{1}{4} \text{ of } 3\tfrac{2}{3} \times \frac{4}{7} \text{ of } \frac{2}{11} = \frac{\cancel{21}^{3}}{\cancel{4}_{1}} \times \frac{\cancel{11}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{7}_{1}} \times \frac{2}{\cancel{11}_{1}} = 2 \text{ } \textit{Ans.}$$

Ex. 6. $\frac{5}{11}$ of $1\frac{1}{2}$ of $\frac{3}{5}$ of $7 \times 4\frac{1}{2}$ of $\frac{4}{15}$ of $1\frac{3}{7}$.

$$= \frac{\cancel{5}^{1}}{11} \times \frac{\cancel{3}^{1}}{\cancel{2}_{1}} \times \frac{8}{\cancel{5}_{1}} \times \frac{\cancel{7}^{1}}{1} \times \frac{\cancel{9}^{3}}{\cancel{2}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{15}_{\cancel{3}_{1}}} \times \frac{10}{\cancel{7}_{1}} = \frac{80}{11} = 7\tfrac{3}{11} \text{ } \textit{Ans.}$$

Or we may arrange the numerators and denominators thus,—

$$= \frac{\cancel{5}. \cancel{3}. 8. \cancel{7}. \cancel{9}. \cancel{4}. 10.}{11. \cancel{2}. \cancel{5}. 1. \cancel{2}. \cancel{15}_{\cancel{3}}. \cancel{7}.} = \frac{80}{11} = 7\tfrac{3}{11} \text{ } \textit{Ans.}$$

Ex. 7. Multiply $\frac{3}{14} + \frac{7}{9} + \frac{2}{3} - \frac{1}{18}$ by $\frac{27}{303} - \frac{17}{202}$.

$$\frac{3}{14} + \frac{7}{9} + \frac{2}{3} - \frac{1}{18} = \frac{27 + 98 + 84 - 7}{126} = \frac{202}{126};$$

$$\frac{32}{303} - \frac{17}{202} = \frac{64 - 51}{606} = \frac{13}{606}; \text{ and } \frac{\cancel{202}^{1}}{126} \times \frac{13}{\cancel{606}_{3}} = \frac{13}{378} \text{ } \textit{Ans.}$$

Ex. 28.

1. $\frac{1}{2} \times \frac{3}{5}$.
2. $\frac{3}{7} \times \frac{2}{5}$.
3. $\frac{5}{11} \times \frac{4}{7}$.
4. $\frac{3}{4} \times \frac{2}{3}$.
5. $\frac{1}{4} \times \frac{3}{5}$.
6. $\frac{3}{5} \times \frac{3}{5}$.
7. $\frac{2}{3} \times \frac{1}{5}$.
8. $\frac{3}{5} \times \frac{2}{5}$.
9. $\frac{1}{5} \times \frac{5}{11}$.
10. $\frac{3}{5} \times \frac{5}{16}$.
11. $\frac{7}{13} \times \frac{9}{14}$.
12. $\frac{3}{5} \times \frac{2}{15}$.
13. $\frac{1}{5}$ of $\frac{3}{4} \times \frac{1}{15}$.
14. $\frac{11}{14} \times \frac{5}{33}$ of $\frac{7}{15}$.
15. $7 \times \frac{5}{11}$ of $\frac{33}{49}$.
16. $3\frac{1}{3} \times \frac{4}{15}$.
17. $3\frac{1}{3} \times \frac{1}{7}$ of $\frac{1}{15}$.
18. $1\frac{1}{2}$ of $\frac{4}{5} \times 7\frac{1}{2}$.

19. $\frac{3}{5}$ of $\frac{7}{15} \times \frac{4}{11}$ of $\frac{10}{11}$.
20. $\frac{8}{11}$ of $\frac{5}{12} \times 1\frac{1}{10}$ of $3\frac{1}{3}$.
21. $3\frac{1}{2}$ of $\frac{1}{14}$ of $3\frac{1}{3} \times 19$.
22. $6\frac{2}{7}$ of $8\frac{3}{4} \times \frac{14}{30} \times \frac{14}{47}$ of $1\frac{19}{11}$.
23. $\frac{13}{65}$ of $\frac{33}{91}$ of $\frac{14}{117} \times \frac{19}{195}$ of 39.
24. $\frac{76}{195}$ of $\frac{95}{195} \times 1\frac{13}{15}$ of $\frac{24}{77}$.
25. $2\frac{15}{17}$ of $3\frac{19}{24}$ of $3\frac{1}{13} \times \frac{85}{147}$.
26. $7\frac{1}{4}$ of $\frac{1}{17} \times \frac{5}{87}$ of $6\frac{3}{5}$.
27. $4\frac{5}{7}$ of $7\frac{8}{11}$ of $\frac{14}{65} \times 9\frac{1}{17}$ of $\frac{13}{37}$ of $1\frac{7}{11}$ of $\frac{1}{7}$.
28. $4\frac{3}{5}$ of $1\frac{7}{9}$ of $3\frac{1}{7}$ of $1\frac{33}{65} \times \frac{9}{35}$ of $1\frac{1}{3}$ of $\frac{1}{15}$.
29. $2\frac{41}{105}$ of $1\frac{171}{247}$ of $1\frac{9}{147} \times 10\frac{3}{14}$ of $\frac{3}{343}$ of $1\frac{13}{14}$ of $2\frac{1}{3}$.
30. Multiply $\frac{3}{4} + \frac{3}{5} + \frac{5}{13}$ by $4\frac{3}{7}$ of $1\frac{9}{47}$ of $\frac{1}{4}$.
31. Multiply $3\frac{1}{3} + \frac{5}{8} + \frac{7}{11} + \frac{5}{18}$ by $1\frac{13}{66} - 1\frac{3}{11} + \frac{5}{33}$.
32. Multiply $3\frac{1}{4}$ of $\frac{3}{5} + \frac{17}{34} + 2\frac{1}{6}$ of $3\frac{3}{4}$ by $1\frac{1}{3} - \frac{1}{8} + \frac{1}{4}$.

The fractions $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{8}$, are respectively called the ***reciprocals*** of the fractions $\frac{3}{1}$, $\frac{5}{1}$, $\frac{8}{1}$. Also since $4 = \frac{4}{1}$, $\frac{1}{4}$ is the reciprocal of 4. Similarly, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{12}$, are the reciprocals of 5, 8, 12, respectively. The reciprocal of a fraction is therefore formed by inverting the fraction.

To divide any number by a fraction is equivalent to multiplying it by the reciprocal of the fraction.

For since 1 contains $\frac{1}{3}$ *twice*, therefore 4 contains $\frac{1}{3}$ *four times twice*, or 8 times; that is, $4 \div \frac{1}{3} = 8$; but $4 \times \frac{3}{1} = \frac{12}{1} = 8$. $\therefore 4 \div \frac{1}{3} = 4 \times \frac{3}{1}$. Again, $\frac{2}{3}$ contains $\frac{1}{3}$ three times, or $\frac{2}{3} \div \frac{1}{3} = 3$; but $\frac{2}{3} \times 5 = \frac{10}{3} \times \frac{1}{5} = \frac{10}{15} = 3$. $\therefore \frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{5}{1}$. Hence the rule for division of fractions.

DIVISION.

49. RULE. Invert the divisor and proceed as in multiplication.

Ex. 1. $2 \div \frac{3}{4} = \frac{2}{1} \times \frac{4}{3} = \frac{8}{3} = 2\frac{2}{3}$ *Ans.*

Ex. 2. $\frac{3}{5} \div \frac{21}{30} = \frac{3}{5} \times \frac{30}{21} = \frac{90}{105} = \frac{6}{7}$ *Ans.*

Ex. 3. $\frac{7}{8} \div \frac{35}{36} = \frac{\cancel{7}}{\cancel{8}_{2}} \times \frac{\overset{9}{\cancel{36}}}{\underset{5}{\cancel{35}}} = \frac{9}{10}$ *Ans.*, by cancelling.

Mixed numbers must be reduced to improper fractions before the rule can be applied.

Ex. 4. $\frac{3}{14} \div 2\frac{3}{4} = \frac{3}{14} \div \frac{11}{4} = \frac{3}{\cancel{14}_{7}} \times \frac{\overset{2}{\cancel{4}}}{11} = \frac{6}{77}$ *Ans.*

Ex. 5. $\frac{3}{5}$ of $\frac{4}{9} \div \frac{5}{6}$ of $\frac{8}{11} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{5}}} \times \frac{11}{\underset{1}{\cancel{8}}} = \frac{11}{25}$ *Ans.*

Or thus, $\frac{3}{5}$ of $\frac{4}{9} \div \frac{5}{6}$ of $\frac{8}{11} = \frac{\overset{1}{\cancel{3}}. \overset{1}{\cancel{4}}. \overset{2}{\cancel{6}}. 11}{\underset{1}{\cancel{5}}. \underset{3}{\cancel{9}}. \underset{1}{\cancel{5}}. \underset{1}{\cancel{8}}} = \frac{11}{25}$ *Ans.*

Ex. 6. Divide $3\frac{3}{5}$ of $\frac{11}{10}$ of $2\frac{3}{7}$ by $1\frac{6}{11}$ of $1\frac{1}{3}$ of $\frac{33}{56}$ of $1\frac{7}{13}$.

$$= \frac{\overset{1}{\cancel{17}}. \overset{1}{\cancel{11}}. \overset{4}{\cancel{16}}. 11. \overset{1}{\cancel{3}}. \overset{19}{\cancel{95}}. 13}{\underset{1}{\cancel{5}}. \underset{1}{\cancel{19}}. 7. \underset{1}{\cancel{17}}. \underset{1}{\cancel{4}}. \underset{3}{\cancel{33}}. \underset{1}{\cancel{20}}} = \frac{143}{35} = 4\frac{3}{35}$$ *Ans.*

Ex. 29.

1. $\frac{1}{2} \div \frac{1}{3}$.
2. $\frac{1}{3} \div \frac{1}{2}$.
3. $\frac{2}{5} \div \frac{4}{5}$.
4. $\frac{3}{4} \div \frac{3}{5}$.
5. $\frac{9}{7} \div \frac{3}{14}$.
6. $\frac{14}{15} \div \frac{7}{9}$.
7. $5 \div \frac{5}{8}$.
8. $2\frac{1}{4} \div \frac{3}{5}$.
9. $1\frac{1}{2} \div 1\frac{1}{3}$.
10. $2\frac{3}{11} \div 1\frac{5}{33}$.
11. $\frac{17}{10} \div 4\frac{3}{4}$.
12. $\frac{37}{40} \div 1\frac{1}{9}$.
13. $\frac{9}{5}$ of $1\frac{3}{4} \div 1\frac{1}{2}$.
14. $6\frac{2}{3} \div 1\frac{1}{2}$ of $\frac{7}{4}$.
15. $1\frac{1}{7} \div 3\frac{1}{3}$ of $5\frac{1}{4}$.
16. $\frac{2}{7}$ of $\frac{3}{11} \div \frac{9}{10}$ of $\frac{4}{33}$.
17. $\frac{3}{4}$ of $\frac{3}{5} \div \frac{5}{6}$ of $\frac{1}{7}$.
18. $2\frac{10}{11}$ of $\frac{7}{6} \div \frac{4}{11}$ of $3\frac{1}{6}$.
19. $1\frac{3}{8}$ of $2\frac{2}{3}$ of $4 \div 1\frac{1}{7}$ of $\frac{1}{13}$.
20. $\frac{3}{4}$ of $2\frac{1}{3}$ of $\frac{7}{5} \div 1\frac{1}{5}$ of $1\frac{1}{9}$ of $\frac{1}{4}$.
21. $1\frac{1}{2}$ of $\frac{7}{13}$ of $1\frac{1}{8} \div 5\frac{1}{4}$ of $7\frac{1}{2}$.
22. $\frac{1}{67}$ of $\frac{1}{15} \div 7\frac{1}{3}$ of $\frac{201}{1344}$.
23. $\frac{1}{17}$ of $2\frac{2}{5}$ of $7\frac{2}{7} \div 2\frac{6}{25}$ of $5\frac{1}{11}$.
24. $114 \div \frac{5}{6}$ of $3\frac{1}{6}$.
25. Divide $\frac{3}{4} + \frac{3}{5}$ by $\frac{3}{5} - \frac{1}{4}$.
26. Divide $2\frac{2}{7} + 1\frac{1}{4}$ by $8\frac{1}{4}$.
27. $\frac{1}{33}$ of $4\frac{3}{11}$ of $\frac{35}{201} \div \frac{35}{67}$ of $4\frac{3}{25}$ of $\frac{7}{88}$ of $\frac{1}{3}$.
28. $1\frac{1}{17}$ of $2\frac{3}{14}$ of $1\frac{1}{33} \div \frac{33}{98}$ of $2\frac{9}{65}$ of $16\frac{5}{7}$ of $\frac{1}{27}$.
29. Divide $\frac{3}{8} + \frac{5}{6} + \frac{7}{4} + \frac{5}{6} + \frac{1}{10}$ by $2\frac{1}{4} - 1\frac{3}{4} + \frac{7}{12}$.
30. Divide $4\frac{1}{4} - 2\frac{3}{8} + 3\frac{1}{2} - \frac{4}{35}$ by $11\frac{1}{2} - 9\frac{5}{18} - \frac{11}{24}$.
31. $1\frac{14}{33}$ of $2\frac{4}{7}$ of $\frac{70}{1309} \div 1\frac{19}{33}$ of $2\frac{1}{45}$ of $\frac{35}{216}$ of $\frac{405}{544}$ of $2\frac{1}{9}$.
32. Divide $1\frac{3}{11} + 2\frac{5}{10} + \frac{10}{33}$ by $\frac{33}{40} + 3\frac{5}{32} - 3\frac{1}{60}$.

Complex Fractions.

50. Since the numerator and denominator of a fraction stand in the relation of dividend and divisor, the complex fraction $\frac{2\frac{1}{3} \text{ of } 3\frac{3}{4}}{7\frac{1}{2} \text{ of } \frac{8}{9}}$ is consequently equivalent to $2\frac{1}{3}$ of $3\frac{3}{4} \div 7\frac{1}{2}$ of $\frac{8}{9}$. This will sufficiently explain the method employed in the following examples.

Ex. 1. $\frac{3\frac{3}{4}}{5} = 3\frac{3}{4} \div 5 = \frac{15}{4} \div \frac{5}{1} = \frac{15}{4} \times \frac{1}{5} = \frac{3}{4}$ *Ans.*

Ex. 2. $\frac{\frac{3}{4}}{6\frac{3}{4}} = \frac{3}{4} \div 6\frac{3}{4} = \frac{3}{4} \div \frac{27}{4} = \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{27}_{9}} = \frac{1}{9}$ *Ans.*

Or thus, $\frac{\frac{3}{4}}{6\frac{3}{4}} = \frac{3}{27}$ (multiplying both numerator and denominator by 4) $= \frac{1}{9}$ *Ans.*

Ex. 3. $\frac{2\frac{2}{3} \text{ of } 3\frac{3}{7}}{7\frac{1}{2} \text{ of } \frac{8}{9}} = 2\frac{2}{3} \text{ of } 3\frac{3}{4} \div 7\frac{1}{2} \text{ of } \frac{8}{9}.$

$$= \frac{\cancel{8}}{\cancel{3}} \times \frac{\cancel{15}}{\cancel{4}_{2}} \times \frac{\cancel{2}}{\cancel{15}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{8}} = \frac{3}{2} = 1\frac{1}{2} \text{ Ans.}$$

Ex. 4. $\frac{6\frac{1}{4} + 7\frac{3}{5}}{7\frac{1}{4} - 6\frac{3}{5}} = \frac{\frac{25}{4} + \frac{38}{5}}{\frac{29}{4} - \frac{33}{5}} = \frac{125 + 152}{145 - 132}$ (multiplying numerator and denominator by 20) $= \frac{277}{13} = 21\frac{4}{13}$ *Ans.*

Ex. 5. $\frac{4\frac{1}{2} \times 3\frac{2}{3} - 4}{4\frac{1}{2} \times 3\frac{2}{3} + 4} = \frac{\frac{9}{2} \times \frac{11}{3} - 4}{\frac{9}{2} \times \frac{11}{3} + 4} = \frac{\frac{33}{2} - 4}{\frac{33}{2} + 4}.$

$$= \frac{\frac{33}{2} - \frac{8}{2}}{\frac{33}{2} + \frac{8}{2}} = \frac{\frac{25}{2}}{\frac{41}{2}} = \frac{25}{41} \text{ Ans.}$$

Ex. 6. $\frac{5\frac{4}{7} \text{ of } 1\frac{1}{12}}{1\frac{5}{16} \text{ of } 9\frac{1}{8}} + \frac{3\frac{4}{7} \text{ of } 4\frac{1}{8} \text{ of } 3\frac{1}{5}}{1\frac{1}{33} \text{ of } 2\frac{3}{5} \text{ of } \frac{9}{14}}.$

$$= \frac{5\frac{4}{7} \text{ of } 1\frac{1}{12}}{1\frac{5}{16} \text{ of } 9\frac{1}{8}} \times \frac{1\frac{1}{33} \text{ of } 2\frac{3}{5} \text{ of } \frac{9}{14}}{3\frac{4}{7} \text{ of } 4\frac{1}{8} \text{ of } 3\frac{1}{5}}.$$

$$= \frac{\overset{3}{\cancel{39}}.\ \cancel{11}.\ \cancel{16}.\ \cancel{3}}{\cancel{7}.\ \cancel{12}_{4}.\ \cancel{15}_{5}.\ 28} \times \frac{\overset{17}{\cancel{34}}.\ \overset{1}{\cancel{4}}}{\cancel{33}_{3}.\ \overset{4}{\cancel{20}}.\ 9.} \times \frac{\cancel{9}.\ \cancel{7}.\ \cancel{3}.\ \cancel{5}}{\cancel{14}_{7}.\ \cancel{25}_{5}.\ \cancel{12}.\ \cancel{16}} = \frac{17}{980} \text{ Ans.}$$

Ex. 30.

Reduce to their simplest forms—

1. $\frac{\frac{3}{5}}{\frac{4}{7}}$; $\frac{7}{\frac{1}{6}}$; $\frac{3\frac{1}{2}}{12}$; $\frac{6}{10\frac{1}{2}}$; $\frac{4\frac{2}{3}}{5\frac{1}{4}}$; $\frac{\frac{7}{9}}{9\frac{1}{3}}$; $\frac{\frac{4}{5}}{\frac{7}{10}}$; $\frac{11}{3\frac{1}{3}}$; $\frac{17\frac{1}{2}}{43\frac{3}{4}}$; $\frac{1\frac{3}{11}}{\frac{12}{110}}$.

2. $\frac{1\frac{1}{4}}{2\frac{2}{5}}$; $\frac{7\frac{1}{2}}{1\frac{2}{15}}$; $\frac{31\frac{1}{2}}{2\frac{1}{6}}$; $\frac{2\frac{1}{10}}{3\frac{9}{25}}$; $\frac{6}{\frac{7}{8}}$; $\frac{2\frac{1}{10}}{\frac{13}{17}}$; $\frac{11}{8\frac{1}{16}}$.

3. $\frac{3\frac{6}{7} \text{ of } 1\frac{13}{16}}{2\frac{1}{24} \text{ of } \frac{24}{35}}$; $\frac{1\frac{3}{15} \text{ of } 2\frac{1}{4}}{2\frac{4}{15} \text{ of } 1\frac{1}{8}}$; $\frac{2\frac{3}{24} \text{ of } 3\frac{1}{8}}{4\frac{3}{11} \text{ of } \frac{1}{10}}$; $\frac{\frac{5}{8} \text{ of } \frac{7}{9}}{\frac{3}{8} \text{ of } \frac{2}{7}}$.

4. $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}-\frac{1}{3}}$; $\frac{\frac{1}{6}-\frac{1}{8}}{\frac{1}{4}+\frac{1}{8}}$; $\frac{2\frac{3}{4}-1\frac{1}{6}}{4\frac{1}{12}+\frac{5}{16}}$; $\frac{3\frac{1}{2}+\frac{1}{7}}{1\frac{3}{14}+\frac{1}{16}}$.

5. $\frac{\frac{4}{5} \text{ of } 3\frac{3}{4} \text{ of } \frac{11}{14}}{\frac{3}{8} \text{ of } 3\frac{3}{4}-1\frac{7}{8}}$; $\frac{\frac{7}{8}+\frac{9}{10}+\frac{5}{24}}{\frac{5}{8} \text{ of } 2\frac{2}{3} \text{ of } \frac{11}{20}}$; $\frac{2\frac{1}{2}\times 2\frac{1}{4}-1}{2\frac{1}{4}\times 2\frac{1}{4}+1}$.

6. $\frac{\frac{5}{6} \text{ of } \frac{7}{8}}{\frac{15}{16} \text{ of } 1\frac{3}{4}} \times \frac{7\frac{1}{2}}{\frac{5}{8}}$; $\frac{3\frac{1}{3} \text{ of } \frac{5}{12}}{3\frac{3}{4} \text{ of } \frac{7}{8}}+\frac{3}{\frac{5}{8}}$; $\frac{\frac{7}{11} \text{ of } 1\frac{1}{2}}{2\frac{1}{7} \text{ of } 2\frac{5}{24}}+4\frac{1}{3}$.

Second Course.

1. Divide a quarter by two and a half.

2. Multiply the sum of $3\frac{5}{11}$ and $4\frac{1}{3}$ by their difference, and divide the product by $46\frac{5}{11}$ of $\frac{7}{13\frac{1}{3}}$.

3. Divide the greater of the fractions $\frac{7\frac{1}{4}}{\frac{11}{21}}$ and $\frac{3\frac{1}{2\frac{1}{3}}}{1\frac{3}{4}}$ by the less.

4. A pays $\frac{1}{3}$ of his money to B, $\frac{2}{5}$ of what remains to C, and then he has £2 left. How much had he at first?

5. Divide the sum of $3\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{9}$, $1\frac{1}{12}$, $\frac{9}{10}$, $2\frac{1}{3}$, $\frac{4}{15}$, and $3\frac{1}{3}$, by the sum of $1\frac{1}{3}$, $\frac{5}{8}$, $2\frac{1}{4}$, $\frac{11}{20}$, $\frac{17}{24}$, and $\frac{5}{27}$.

6. What number must be added to $\frac{\frac{7}{12}}{8}\times\frac{4\frac{1}{4}}{\frac{17}{20}}$ to make it equal to the quotient of $2\frac{2}{9}$ of $\frac{13}{18}$ by $3\frac{1}{4}$ of $\frac{1}{8}$ of $10\frac{2}{3}$?

7. A 3-gallon cask contains $2\frac{1}{4}$ gallons of wine and the remainder water; a gallon of the mixture is taken out and put into a 2-gallon cask, which is then filled up with water. What fraction of the contents of the second cask is wine?

8. Suppose a cannon ball goes a mile the first second, half a mile the second second, a quarter of a mile the third second, &c.; how much short of two miles will it be after 10 seconds?

9. $\frac{3\frac{1}{2} \text{ of } 4\frac{1}{2}}{5\frac{2}{3} \text{ of } 1\frac{1}{7}}+\frac{5\frac{1}{7} \text{ of } 2\frac{2}{5}}{4\frac{2}{3} \text{ of } 3\frac{3}{11}}+\frac{7\frac{3}{5} \text{ of } 1\frac{3}{13}}{3\frac{3}{4} \text{ of } 6\frac{2}{3}}+\frac{\frac{1}{5}+\frac{2}{5}}{\frac{1}{3} \text{ of } \frac{5}{6}}$.

10. Divide $\frac{13\frac{1}{5} \text{ of } 1\frac{5}{11} \text{ of } 13\frac{3}{5}}{\frac{33}{55} \text{ of } 1\frac{4}{13} \text{ of } 2\frac{3}{10}}$ by $\frac{\frac{27}{55} \text{ of } \frac{33}{37} \text{ of } 1\frac{7}{29}}{\frac{14}{17} \text{ of } \frac{33}{35} \text{ of } 1\frac{5}{37} \text{ of } 22\frac{3}{32}}$.

11. The sum of two numbers is $37\frac{1}{31}$, the lesser number is $14\frac{17}{46}$; find the greater number.

12. The sum of two numbers is $11\frac{4}{11}$, and their difference is $3\frac{5}{14}$; find the numbers.

13. A has $\frac{7}{15}$ of a ship, and B the remainder; C buys $\frac{3}{7}$ of A's share, and $\frac{2}{5}$ of B's; what is C's share after the transaction?

14. The product of two numbers is equal to $\frac{5\frac{1}{2}}{2\frac{3}{4}}$; one of the numbers is equal to $\frac{5}{7\frac{1}{2}} - \frac{3}{16}$; find the other number.

15. What number divided by the half of $\frac{11\frac{1}{4}}{9}$ will give $\frac{5}{8}$?

16. An 8-gallon cask is filled with a mixture of $\frac{5}{7}$ wine and the remainder water; a $7\frac{1}{2}$-gallon cask is filled with $\frac{3}{5}$ wine and the remainder water; and a $4\frac{1}{2}$-gallon cask is filled with $\frac{1}{3}$ water and the remainder wine. They are all emptied into a larger vessel, and a 12-gallon cask is filled with the mixture. How many gallons of wine does the 12-gallon cask contain?

DECIMAL FRACTIONS.

51. Decimal Fractions are formed by a simple extension of the common system of notation. In the number 542, the 5 represents 5 *hundreds*, the 4 represents 4 *tens*, and the 2 represents 2 *units;* that is, the value of any figure decreases tenfold for every successive place it is moved towards the right. If we continue to affix figures to the right of the unit's place, then, by the same method of notation, the first figure on the right of the unit's place will represent *tenths*, the next, *hundredths*, the next, *thousandths*, &c. A point called the *decimal point* is placed after the unit's figure to distinguish the whole numbers on the left from the fractional parts on the right. The following table exhibits the value of each figure accord*ing to* its position.

&c.	thousands.	hundreds.	tens.	*units.*	tenths.	hundredths.	thousandths.	ten thousandths.	&c.
	3	0	4	7 ·	6	1	0	9	

The above figures represent $3000+40+7+\frac{6}{10}+\frac{1}{100}+\frac{0}{1000}+\frac{9}{10000}$, and this, by addition of vulgar fractions, $=3047\frac{6109}{10000}$, or $\frac{30476109}{10000}$. Similarly, $4{\cdot}01=4\frac{1}{100}$, or $4\frac{1}{100}$; $\cdot0073=\frac{73}{10000}$.

The following distinctions will now be understood,—

(1.) A vulgar fraction may have any number for its denominator; but the denominator of a decimal fraction must be 10, or some power of 10, that is, it must be 10, or 100, or 1000, &c.

(2.) In a vulgar fraction the denominator must be expressed; but in a decimal fraction the necessity for writing the denominator is obviated by the use of the decimal point.

The following rule for the conversion of a decimal into a vulgar fraction, is evident from what has already been stated.

52. RULE. Write the decimal itself for the numerator, and for the denominator write 1, followed by as many 0's as there are figures after the point.

Ex. 1. Express 24·7; ·003; 604·0503, as vulgar fractions.

$24{\cdot}7=\frac{247}{10}$, or $24\frac{7}{10}$; $\cdot003=\frac{3}{1000}$; $604{\cdot}0503=\frac{6040503}{10000}$.

When any figures stand before the point it is best to write the result as a mixed number; thus, $604{\cdot}0503=604\frac{503}{10000}$.

Ex. 2. Express ·25; 3·128; and ·0375, as vulgar fractions in their lowest terms.

$\cdot25=\frac{25}{100}=\frac{1}{4}$; $3{\cdot}128=3\frac{128}{1000}=3\frac{16}{125}$; $\cdot0375=\frac{375}{10000}=\frac{3}{80}$.

Conversely, any vulgar fraction whose denominator is 10, or any power of 10, may be expressed as a decimal by writing the numerator for the decimal, and marking off as many figures to the right of the point as there are 0's in the denominator, prefixing 0's if there are not sufficient figures.

Ex. 3. Express $\frac{3}{10}$, $\frac{17}{10}$, $\frac{7}{100}$, and $\frac{33}{10000}$, as decimals.

$\frac{3}{10} = \cdot 3$; $\frac{17}{10} = 1 \cdot 7$; $\frac{7}{100} = \cdot 07$; $\frac{33}{10000} = \cdot 0033$.

53. RULE. *To multiply a decimal by* 10, *or any power of* 10,—Move the decimal point as many places to the *right* as there are 0's in the multiplier; if there are not figures sufficient, affix 0's to make up the number.

Conversely, *to divide a decimal by* 10, *or its powers*,—Move the point as many places to the *left* as there are 0's in the divisor; if there are not figures sufficient, prefix 0's to make up the number.

For $7 \cdot 19 = \frac{719}{100}$, and $\frac{719}{100} \times 10 = \frac{719}{10} = 71 \cdot 9$.
Conversely, $71 \cdot 9 \div 10 = 7 \cdot 19$.

Examples.

(1). $27 \cdot 35 \times 10 = 273 \cdot 5$.
(2). $27 \cdot 35 \times 100 = 2735$.
(3). $27 \cdot 35 \times 10000 = 273500$.
(4). $\cdot 027 \times 100 = 2 \cdot 7$.
(5). $35 \cdot 7 \div 10 = 3 \cdot 57$.
(6). $6 \cdot 19 \div 1000 = \cdot 00619$.
(7). $93 \div 100 = \cdot 93$.
(8). $230 \div 1000 = \cdot 23$.

In connexion with these examples it should be observed, that the value of a decimal is altered by prefixing 0's, but not by affixing them. $\cdot 3 = \frac{3}{10}$, but $\cdot 003 = \frac{3}{1000}$. Again, $\cdot 7 = \frac{7}{10}$, and $\cdot 70 = \frac{70}{100}$, which in its lowest terms also $= \frac{7}{10}$. Hence, 0's are never written on the right of a decimal. See Example 8.

Ex. 31.

1. Express $\cdot 3$; $\cdot 71$; $\cdot 09$; $\cdot 307$; $\cdot 00019$; $5 \cdot 11$; $29 \cdot 7$; $\cdot 001$; $314 \cdot 073$; and $\cdot 0030001$, as vulgar fractions.

2. Express $\frac{21}{100}$, $\frac{19}{10000}$, $\frac{501}{10}$, $39\frac{3}{100}$, as decimals.

3. Express $\cdot 5$; $\cdot 15$; $\cdot 625$; $\cdot 48$; $4 \cdot 5$; $31 \cdot 0025$; and $\cdot 0875$, as vulgar fractions in their lowest terms.

4. Multiply $397 \cdot 41$ by 10, 100, and 100000, separately.

5. Divide 41 13 by 10, 1000, and a million, separately.

6. Express $517\frac{3}{1000}$ as a decimal, and multiply it by 10000.

7. Multiply $\cdot 3125$ by 10, 100, and 1000, separately, and express the results as vulgar fractions in their lowest terms.

8. Divide 241875 by 1000, 100000, and ten millions, separately, and express the results as vulgar fractions in their lowest terms.

ADDITION.

54. RULE. Arrange the numbers in a column, so that the points may all be in the same line, and proceed as in simple addition, placing the point in the sum directly under the other points.

Ex. 1. Add together 34·4; ·09; 3·701; and 217·3.

Ex. 2. 15·9 + ·314 + ·007 + 24·08 + 692·5 + ·009.

Ex. 3. ·0007 + 3·9 + 68 + 2914·7 + ·710003 + 19.

(1.)	(2.)	(3.)
34·4	15·900	·0007
·09	·314	3·9
3·701	·007	68·
217·3	24·080	2914·7
255·491 *Ans.*	692·500	·710003
	·009	19
	732·81	3006·310703

The correctness of this operation may be shown by converting the decimals into vulgar fractions, and performing the addition in that form. Ex. 1. becomes $34\frac{4}{10} + \frac{9}{100} + 3\frac{701}{1000} + 217\frac{3}{10} = 34\frac{400}{1000} + \frac{90}{1000} + 3\frac{701}{1000} + 217\frac{300}{1000} = 34 + 3 + 217 + \frac{400+90+701+300}{1000} = 254\frac{1491}{1000} = 255\frac{491}{1000} = 255{\cdot}491$.

We may fill the empty spaces with 0's as in Ex. 2., or leave them blank as in Ex. 3.

Observe, that in the answer to Ex. 2. the final 0 is left out.

Ex. 32.

1. 29·1 + 36·32 + 14·293 + ·06 + 60·8 + 5·555 + 4·2.
2. ·006 + 33·2 + 171·89 + ·0004 + ·05 + 631·1 + 9·04.
3. 417·0361 + 64·9 + ·05067 + 18 + 3·4 + 127·0091.
4. 6·12345 + ·04001 + 29 + ·00994 + 3651·2.
5. 3891·216 + 48·9 + ·967 + 23·03 + ·941 + 30.
6. ·31456 + ·00131 + 1·25164 + ·43249.
7. 354·069 + 9·8 + 777·77 + ·60041 + ·8 + 67.
8. 98·765 + 5879·6 + 834·712 + 99·46738 + ·8794.
9. 436·781 + 7·132 + ·0045 + ·00613 + 86·07637.
10. Nine tenths + seventy-seven hundredths + seven ten-thousandths + eighty-six hundredths + four thousand two hundred and eighty-three ten-thousandths.
11. ·1 + ·004 + ·05368 + ·129793 + ·244437 + ·98709 + ·181 + ·2.

12. Add together thirty-six thousandths; fifty-three millionths; six hundred and seventy-nine hundred-thousandths; eight hundred and ninety-six tenths; seven thousand-millionths; and six million eight hundred and two thousandths.

SUBTRACTION.

55. RULE. Place the lesser number under the greater, so that the point of the lesser one is under the point of the greater, and proceed as in simple subtraction, placing the point in the result under the others.

Exs. 43·4−7·29; ·7−·6134; 490−·312.

(1).		(2).		(3).	
43·40		·7		490·000	
7·29		·6134		·312	
36·11	*Ans.*	·0866	*Ans.*	489·688	*Ans.*

The proof of this rule is similar to that for addition of decimals. Thus, in Ex. (1) $43{\cdot}4-7{\cdot}29=43\frac{4}{10}-7\frac{29}{100}=36\frac{11}{100}=36{\cdot}11$.

We may either fill up the blank spaces with 0's as in Exs. (1) and (3), or leave them vacant as in Ex. (2).

Ex. 33.

1. 51·32−4·27.
2. 634·012−14·371.
3. ·8176−·349.
4. ·009−·007134.
5. 63−57·9876.
6. 42·3145−41·6475.
7. 3214·02−97·987.
8. 23·41763−22·30854.
9. 8600−·99993.
10. 3417·5−6·1734892.
11. Three, and twenty-one thousandths−eight hundredths.
12. From the sum of 31·47; 2·13; 6·94; 7·831; ·0009; and 314·206; take the sum of 217·83; 12·72; 9·631; ·7843; 118·6; and ·91189.

MULTIPLICATION.

56. RULE. Multiply as in whole numbers, and mark off in the product as many decimal figures to the right of the point as there are in the multiplier and multiplicand together; if there are not figures sufficient, prefix 0's to make up the number.

Ex. 1. Multiply 3·24 by 21·6: and 3·1 by ·00043.

(1). 3·24 × 21·6 = 69·984 *Ans.*

(2). 3·1 × ·00043 = ·001333 *Ans.*

In Ex. (1) we have $3{\cdot}24 \times 21{\cdot}6 = 3\frac{24}{100} \times 21\frac{6}{10}$.

$= \frac{324}{100} \times \frac{216}{10} = \frac{69984}{1000} = 69{\cdot}984$.

In (1) we have two decimal figures in the multiplicand, and one in the multiplier, and therefore three in the product.

In (2) we have one in the multiplicand, five in the multiplier, and therefore six in the product, two 0's being prefixed to make up the number.

Ex. 34.

1.	42·3 × 6·11.	2.	31·4 × 2·6.	3.	7·48 × ·314.
4.	·35 × ·74.	5.	21·09 × ·09.	6.	·241 × ·015.
7.	631·77 × ·7498.	8.	·008 × ·0089.	9.	876 × 3·517.
10.	84·93 × 67.	11.	·315 × ·24.	12.	6·25 × 2·56.
13.	816·79 × ·004.	14.	·275 × ·144.	15.	·0002 × ·002.
16.	6·1753 × 819·74.	17.	9163·75 × 8·8.	18.	·7149 × ·0987.
19.	637 × ·0004.	20.	875 × 31·88.	21.	6·457 × 2·09.
22.	421·07 × 3·769.	23.	31·3 × .21.	24.	3·13 × 2·1.

In any series of questions where the *significant* figures are the same in the multipliers and multiplicands, if we obtain one of the answers in the usual way the other answers may be written down at once, without working the remaining questions.

Ex. Multiply 2·13 by ·54; ·213 by 54; ·0213 by 5·4; 213 by ·054; ·00213 by ·0054; and 21·3 by 5·4.

(1). 2·13 × ·54 = 1·1502

(2). ·213 × 54 = 11·502 *Ans.*

(3). ·0213 × 5·4 = ·11502 *Ans.*

(4). 213 × ·054 = 11·502 *Ans.*

(5). ·00213 × .0054 = ·000011502 *Ans.*

(6). 21·3 × 5·4 = 115·02. *Ans.*

It will be seen that we have only to count the decimal figures in the multiplier and multiplicand, and mark off the requisite number in the product.

25. Multiply 61·4 separately by 2·17, by ·217, and by 21·7.

26. Multiply 3·42 by ·018; 34·2 by ·18; and ·342 by 18.

27. Multiply 875 by ·256; ·875 by 2·56; and 87·5 by 25·6.

28. Multiply ·003625 by ·88; 362·5 by ·088; 3625 by 8·8; 3·625 by ·000088; and ·03625 by ·0088.

29. Take nine hundredths from one tenth, and multiply the remainder by ·01.

30. Find the continued product of ·3 × ·003 × 30 × 300.

31. Add together, 134·6; ·457; ·08; 9134·5; 6179·8342; 4·3688; and multiply the sum by 314·65.

32. Multiply 228·83 by 22·5, and express the result as a vulgar fraction in its lowest terms.

DIVISION.

57. *Case I. When the dividend contains a greater number of decimal figures than the divisor.*

RULE. Proceed as in simple division, and mark off in the quotient as many decimal figures as are equal to the excess of those in the dividend over those in the divisor; if there are not figures sufficient, prefix 0's to make up the number.

Ex. 1. Divide 2·20742 by 7·03.

7·03)2·20742(·314 *Ans.*
2109
984
703
2812
2812

For $2 \cdot 20742 \div 7 \cdot 03$.
$= \frac{220742}{100000} \div \frac{703}{100}$.
$= \frac{220742}{100000} \times \frac{100}{703} = \frac{314}{1000} = \cdot 314$.

Ex. 2. 2·46)·005658(·0023 *Ans.*
492
738
738

Ex. 3. ·36)6·3(17·5 *Ans.*
36
270
252
180
180

In Ex. 2, there are six decimal figures in the dividend, two in the divisor, and consequently, 6−2, or 4, in the quotient; but as the division only gives two figures, two 0's are prefixed to make up the required number. In Ex. 3, we have to bring down two 0's before the division terminates: these 0's may or may not be placed after the 3 in the dividend, but they must be reckoned as decimal figures.

58. *Case II. When the number of decimal figures in the divisor exceeds those in the dividend.*

RULE. Affix 0's to the dividend till the number of decimal figures is equal to the number in the divisor: the quotient up to this point in the division is a whole number: if the division does not terminate without bringing down more 0's, the quotient after this point is a decimal.

Ex. 57·6÷·256; 42·6÷3·125.

·256)57·600(225 *Ans.*	3·125)42·600(13·632 *Ans.*
51 2	31 25
6 40	11 350
5 12	9 375
1 280	1 9750
1 280	1 8750
	10000
	9375
	6250
	6250

In the Example on the right we have to bring down three 0's *besides* the two which were affixed, and this makes altogether six decimal figures in the dividend, and consequently there are three in the quotient.

When the number of decimal figures is the same in both dividend and divisor the quotient is, of course, a whole number, if the division terminates without affixing 0's to the dividend.

Sometimes the division will not terminate at all, in which case it is sufficient to carry the quotient to any given number of decimal figures.

In any series of questions where the *significant* figures are alike in the dividends and alike in the divisors, after finding any one of the answers in the usual way, we may deduce the others as in multiplication.

Ex. 35.

1. 2·88÷1·2.	2. 4·16÷·8.	3. ·153÷·9.
4. 33·6÷7.	5. 3·476÷1·1.	6. ·0039÷·13.
7. 6·768÷2·4.	8. 31·74÷10.	9. 7·63÷1000.
10. 1·519÷·49.	11. 45·99÷7·3.	12. ·0092÷2·3.
13. ·77÷3·5.	14. 5·61÷17.	15. ·0031÷1·6.
16. ·9÷12·5.	17. ·1÷6·4.	18. 7·199÷·313.
19. 19·61÷3·7.	20. ·6399÷·711.	21. 413·6÷51·7.
22. ·79÷·32.	23. ·003÷9·6.	24. 1·4÷875.

25. 13·576086÷·4167.
26. 175·3641÷24·39.
27. 167·649÷·235.
28. 52·647÷81·75.

29. 3·7171 + ·0512.
30. ·0033313 + ·00875.
31. 59·63 + ·00089.
32. ·0903 + ·00043.
33. 53·6 + ·00256.
34. 16202·4 + ·314.
35. 486·7 + ·0157.
36. 7866 + ·0184.
37. 471·13194 + 7·19.
38. 50·7 + 9·75.

39. Divide 24·024 separately by ·0312; 85·8; and 8·008.

40. Divide 40360·32 separately by ·2016; 1696·8; and ·04032.

41. Divide 98303·3688 by 2·507739 and by 3343·652.

Work one question in each of the following series, and deduce the remaining answers.

42. Divide 3·23 by ·19; ·0323 by 1·9; 32·3 by 1900; 323 by ·019; 32300 by ·00019; and ·323 by ·0019.

43. Divide 55·89 by ·648; 55890 by 6·48; ·005589 by 648; 558900 by ·00648; and 5589 by ·648.

44. Divide 397 by 1·024; ·0397 by 1024; 3·97 by 10·24; ·00397 by ·001024; and 397000 by ·01024.

Divide correctly to seven decimal figures—

45. 63·71 by 85·3; 6·4 by ·0577; 30 by ·00684.

46. 10·03 by 6897; 9·415 by 3789; 101·01 by ·076.

47. 34017 by ·019; 1 by 31; ·0116 by 73·98.

48. 355 separately by 113, by ·0013, and by 1130000.

Second Course.

1. Express $\frac{13}{1000}$ as a decimal, and multiply it by 100.

2. Express $\frac{2.7}{100}$ + 10000 in a decimal form.

3. Divide 647·39 + 812·5 + 9163 + 7·895 + ·0222 by ·0786.

4. Divide the difference between thirty-nine thousandths and eleven hundredths by the sum of ·00397 and 11891.

5. Find the L.C.M. of 2·4; ·35; 18; ·56; ·048; and 20.

6. Divide the continued product of ·03 × 30 × ·3 × ·003 × 300 by 8·1.

7. Find the G.C.M. of 61·061 and 2445·3.

8. Divide the sum of 42·93; 631·7; and ·9346; by 56·29705.

9. The product of two numbers is ·00351; what would be the product if each of the numbers were a hundred times as great?

10. The quotient in a division is 7·19; what would the quotient be if the dividend had been multiplied by 1000, and the divisor by ·01?

11. What number must be added to 315·71911 so as to make it exactly divisible by ·03147?

12. Add together the sum, difference, product, and quotient of the greater by the less, of ·6641 and ·0725.

59. *To reduce a vulgar fraction to a decimal.*

RULE. Affix decimal cyphers to the numerator, and then divide it by the denominator as in division of decimals.

Ex. 1. Reduce $\frac{1}{8}$; $3\frac{1}{4}$; and $7\frac{5}{8}$, to decimals.

(1). 8)1·000
·125 *Ans.*

(2). $3\frac{1}{4} = \frac{13}{4}$
4)13·00
3·25 *Ans.*

(3). 8)5·000
·625; *Ans.* 7·625.

We may reduce the mixed number to an improper fraction as in (2), but it is better to reduce the fractional part separately as in (3).

Ex. 2. Reduce $\frac{23}{32}$, and $2\frac{1}{2}$ of $\frac{5}{8}$ of $3\frac{1}{4}$ to decimals.

(1). $\frac{23}{32} = \frac{23}{4 \times 8} = {}^{5}\cdot{}^{7}\frac{5}{8} = \cdot71875$ *Ans.*

(2). $2\frac{1}{2} \times \frac{5}{8} \times 3\frac{1}{4} = \frac{5}{2} \times \frac{5}{8} \times \frac{13}{4} = \frac{325}{64} = 5\cdot234375$ *Ans.*

In (1) we first divide by the factor 4 and then by 8. Both examples might have been worked by long division. It is always best to reduce compound fractions to simple ones as in (2).

Ex. 3. Reduce $3\frac{3}{4} + \frac{5}{8} - \frac{13}{16}$; and $\frac{2\frac{1}{2} \text{ of } \frac{7}{8}}{\frac{5}{48} \text{ of } 3\frac{1}{8}}$ to decimals.

(1). $3\frac{3}{4} + \frac{5}{8} - \frac{13}{16} = 3\frac{12+10-13}{16} = 3\frac{7}{16} = 4\frac{7}{16} = 4\cdot0625$ *Ans.*

(2). $\frac{2\frac{1}{2} \text{ of } \frac{7}{8}}{\frac{5}{48} \text{ of } 3\frac{1}{8}} = 2\frac{1}{2} \text{ of } \frac{7}{8} \div \frac{5}{48} \text{ of } 3\frac{1}{8} = \frac{5}{2} \text{ of } \frac{7}{8} \div \frac{5}{48} \text{ of } \frac{25}{8}$

$$= \frac{\overset{3}{\cancel{5}} \times 7 \times \cancel{48} \times 5}{2 \times 8 \times \cancel{5} \times \cancel{25}} = \frac{105}{16} = 6\cdot5625 \text{ } \textit{Ans.}$$

Ex. 4. Reduce $\frac{37}{128}$, and $\frac{5}{16} \times \cdot034$ to decimals.

(1). 128)37·0000000(·2890625 *Ans.*
256
1140
1024
1160
1152
800
768
320
256
640
640

Or by factors thus,—

128 { 4 | 37·0000000
4 | 9·2500000
8 | 2·3125000
·2890625

(2). $\frac{5}{16} \times \cdot034$.
$= \cdot3125 \times \cdot034 = \cdot010625$ *Ans.*

Ex. 36.

Reduce to decimals,—

1. $\frac{1}{4}$; $\frac{3}{4}$; $\frac{5}{8}$; $3\frac{1}{2}$; $7\frac{1}{8}$; $\frac{11}{16}$; $\frac{15}{32}$; $\frac{23}{40}$; and $5\frac{19}{80}$.
2. $\frac{23}{64}$; $\frac{79}{128}$; $29\frac{33}{125}$; $\frac{133}{512}$; $\frac{113}{125}$; and $259\frac{91}{160}$.

3. $\frac{3}{8}$ of $\frac{5}{6}$; $\frac{1}{9}$ of $1\frac{9}{10}$; $2\frac{3}{10}$ of $1\frac{3}{16}$; and $4\frac{1}{7}$ of $5\frac{1}{4}$.

4. $6\frac{1}{4}$ of $1\frac{9}{5}$ of $1\frac{5}{13}$ of $\frac{1}{46}$; and $1\frac{1}{44}$ of $5\frac{1}{11}$ of $3\frac{1}{13}$ of $\frac{88}{315}$.

5. $\frac{1}{2}+\frac{3}{4}+\frac{4}{5}+\frac{6}{7}$. 6. $\frac{7}{8}+\frac{9}{10}+\frac{11}{16}+\frac{13}{20}$. 7. $2\frac{1}{8}+5\frac{4}{5}+\frac{1}{3\cdot5}$.

8. $3\frac{5}{16}+\frac{9}{40}+\frac{7}{13}$ of $\frac{5}{64}$. 9. $\frac{3}{11}$ of $4\frac{2}{5}+\frac{5}{6}$ of $1\frac{1}{2}$. 10. $\frac{13}{24}$ of $2\frac{1}{4}+\frac{57}{1258}$.

11. $\frac{7}{8}+\frac{3}{5\cdot13}+\cdot03$. 12. $7\frac{4}{15}$ of $\frac{23}{40}+\cdot351$. 13. $\frac{17}{10144}-\frac{13}{2856}+\frac{1}{8}$.

14. $5\frac{1}{3}$ of $\frac{32}{44}$ of $4\cdot003$. 15. $\frac{1}{6\cdot3}+1\frac{3}{6\cdot5}$ of $\frac{5}{10}$. 16. $\cdot6+1\frac{47}{49}$ of $1\frac{57}{58}$.

17. $\frac{\cdot076\times13\cdot5}{\cdot95\times\cdot9}$. 18. $\frac{\cdot051 \text{ of } 690}{\cdot85 \text{ of } 2\cdot07}$. 19. $\frac{1\frac{1}{2} \text{ of } 6\cdot09}{11\frac{5}{18} \text{ of } 17\cdot6}$.

20. $\frac{7\frac{1}{8}}{3\frac{3}{4}}\times\cdot004$. 21. $\frac{4\frac{4}{5}}{2\frac{2}{7}}-1\cdot041$. 22. $\frac{\cdot0185\times37\cdot1}{\cdot212\times2\cdot96}$.

60. In reducing a vulgar fraction to a decimal, we have always to affix one or more cyphers to the numerator in order to make it divisible by the denominator; so that the numerator thus becomes a power or multiple of 10. Now, since 10 is only divisible by the factors 2 and 5, it follows that any power of 10 can only be divisible by 2 and 5, or by their powers, singly or combined. Hence, if a fraction be in its lowest terms, and its denominator contains any other factor than 2 and 5, or the powers of these numbers, the division can never terminate. Thus, $\frac{2}{9}=\cdot2222$, &c.; $\frac{3}{11}=\cdot272727$, &c.; $\frac{5}{12}=\cdot41666$, &c.; $\frac{1}{7}=\cdot571428571428571428$, &c. These decimals are respectively written, $\cdot\dot{2}$; $\cdot2\dot{7}$; $\cdot41\dot{6}$; $\cdot\dot{5}71428\dot{5}$. The point over $\cdot\dot{2}$ indicates that the figure repeats; the points over $\cdot2\dot{7}$ indicate the repetition of both figures; in $\cdot41\dot{6}$ only the 6 repeats; and in $\cdot\dot{5}71428\dot{5}$ the whole of the figures repeat.

Decimals of this kind are called *Circulating* or *Repeating decimals*; they are further called *Pure* or *Mixed* circulating decimals, according as the whole or only a part of the decimal figures repeat. The repeating figures taken together are called the *Period*.

$\cdot\dot{4}$; $\cdot2\dot{3}$; $6\cdot754\dot{1}$, are pure circulating decimals.

$\cdot014\dot{3}$; $5\cdot67\dot{2}$; $\cdot23147\dot{}$, are mixed circulating decimals.

61. *To reduce a pure circulator to a vulgar fraction.*

RULE. Write the decimal itself for the numerator, and as many 9's as there are figures in the period for the denominator.

Ex. 1. $\cdot\dot{7}=\frac{7}{9}$. *Ex.* 2. $\cdot\dot{4}3\dot{2}=\frac{432}{999}=\frac{16}{37}$ in its lowest terms.

Ex. 3. $47\cdot0\dot{8}1=47\frac{81}{999}=47\frac{3}{37}$ in its lowest terms.

The reason for this rule may be thus shown,—

$$\begin{array}{rrcl} & \cdot7777 \text{ \&c.} & \times & 10 = 7\cdot7777 \\ & \cdot7777 \text{ \&c.} & \times & \underline{1} = \underline{\cdot7777} \\ \text{subtracting,} & \cdot7777 \text{ \&c.} & \times & 9 = 7 \end{array}$$

dividing by 9, $\cdot7777 \text{ \&c.} = \frac{7}{9}$; or, $\cdot\dot{7} = \frac{7}{9}$.

Similarly, in Exs. 2 and 3 we should multiply by 1000 and by 1, and then subtract and divide in the same way.

62. *To reduce a mixed circulator to a vulgar fraction.*

RULE. Consider the whole decimal to the end of the first period, and the non-circulating part as two separate whole numbers: subtract the latter from the former, and make the result the *numerator*. Write as many 9's as there are circulating figures, followed by as many 0's as there are non-circulating figures for the *denominator*.

Ex. 1. $\cdot7\dot{4} = \frac{74-7}{90} = \frac{67}{90}$ *Ans.* *Ex.* 2. $\cdot0\dot{1} = \frac{1}{90}$ *Ans.*

Ex. 3. $\cdot41\dot{6} = \frac{416-41}{900} = \frac{375}{900} = \frac{5}{12}$ in its lowest terms.

Ex. 4. $8\cdot049\dot{0} = 8\frac{490-49}{9000} = 8\frac{441}{9000} = 8\frac{49}{1000}$ *Ans.*

The reason for this rule may be shown by the same process as that employed for the last.

$$\begin{array}{rrcl} & \cdot7444 & \times & 100 = 74\cdot444 \\ & \cdot7444 & \times & \underline{10} = \underline{7\cdot444} \\ \text{subtracting,} & \cdot7444 & \times & 90 = 67 \end{array}$$

dividing by 90, $\cdot7444 \text{ \&c.} = \frac{67}{90}$; or, $\cdot7\dot{4} = \frac{67}{90}$.

In Exs. 3 and 4 we should multiply first by 1000, then by 100, and subtract and divide in the same way. The *general* method of proof is to multiply first by such a power of 10 as will remove the decimal point to the right of the first period; and next, to multiply by such a power of 10 as will place the point between the circulating and non-circulating parts of the decimal.

Ex. 37.

Reduce to circulating decimals,—

1. $\frac{2}{3}$; $\frac{5}{6}$; $\frac{1}{6}$; $\frac{7}{11}$; $\frac{5}{13}$; $\frac{7}{15}$; $\frac{13}{15}$; $\frac{17}{14}$; $\frac{11}{30}$.
2. $2\frac{1}{3}$; $3\frac{5}{6}$; $\frac{9}{7}$; $29\frac{13}{17}$; $\frac{7}{13}$; $\frac{123}{148}$; $\frac{913}{715}$; $4\frac{143}{150}$.
3. $\frac{5}{6}$ of $\frac{7}{13}$; $3\frac{1}{3}$ of $\frac{9}{16}$ of $\frac{5}{11}$; $2\frac{3}{5}$ of $5\frac{7}{11}$ of $8\frac{7}{9}$.

Express as vulgar fractions in their lowest terms,—

4. $\cdot\dot{5}$; $\cdot\dot{7}$; $\cdot\dot{3}$; $\cdot3\dot{1}$; $\cdot7\dot{1}\dot{8}$; $4\cdot6\dot{2}\dot{3}$; $53\cdot20\dot{5}$.

5. $\cdot0\dot{4}$; $\cdot3\dot{4}$; $\cdot00\dot{7}$; $\cdot5\dot{9}$; $\cdot61\dot{3}$; $8\cdot04\dot{9}$; $71\cdot8\dot{8}$.
6. $\cdot638$; $\cdot004\dot{3}2\dot{9}$; $6\cdot37\dot{8}1\dot{2}$; $78\cdot64\dot{9}$; $\cdot00\dot{7}\dot{2}$.
7. $\cdot3\dot{7}0$; $\cdot\dot{7}6923$; $\cdot314\dot{7}8\dot{9}$; $46\cdot829\dot{5}\dot{4}$; $9\cdot\dot{9}$.
8. $\cdot43\dot{7}21628$; $\cdot00\dot{1}9\dot{7}76$; $\cdot43128\dot{1}8\dot{7}$.

By one or other of the two previous rules any circulating decimal may be expressed as a vulgar fraction. And conversely, there can be no decimal which is not the result of the division of the numerator of some fraction by its denominator. The decimal $\cdot\dot{9}$ is apparently an exception; for, according to the rule $\cdot\dot{9}=\frac{9}{9}$, or 1. Now 9+9 will never produce the repeating decimal $\cdot\dot{9}$; but it must be observed that $\cdot\dot{9}$ has no existence except as a mere *assumption*, and can never be the *result* of any arithmetical operation. It is true that $\dot{9}$ can be made to differ from 1 by less than any assignable fraction; and in this sense it is that *any* circulating decimal can be said to be equal to a vulgar fraction.

63. In the addition and subtraction of circulating decimals, it is frequently required to find the sum and difference accurate to a given number of decimal places. In addition, this is accomplished by carrying the operation to two or three more places than the required number. In subtraction, two places beyond the required number will always be sufficient.

The multiplication and division of circulating decimals is best effected by converting the decimals into vulgar fractions, performing the operations in that form, and reducing the result to a decimal.

Exs. Find the value (correct to 5 decimal places) of $\cdot61\dot{2}+4\cdot8\dot{3}2\dot{9}+\cdot\dot{7}+3\cdot35+\cdot32\dot{7}$; and $3\cdot6\dot{1}-2\cdot9\dot{4}\dot{7}$; also $3\cdot\dot{1}\times\cdot8\dot{5}7142\dot{}$; and $8\cdot25+3\cdot\dot{6}$.

(1). $\cdot6122222$
$4\cdot8329832$
$\cdot7777777$
$3\cdot3500000$
$\cdot3272727$
$9\cdot9002558$
Ans., $9\cdot90025$.

(2). $3\cdot6111111$
$2\cdot9479479$
$\cdot6631632$
Ans., $\cdot66316$.

(3). $3\cdot\dot{1}\times\cdot8\dot{5}7142\dot{}$
$=3\frac{1}{3}\times\frac{857142}{999999}$
$=\frac{3}{9}\times\frac{8}{7}=\frac{8}{3}=2\cdot\dot{6}$ *Ans.*

(4). $8\cdot25+3\cdot\dot{6}=8\frac{1}{4}+3\frac{2}{3}$.
$=\frac{33}{4}+\frac{11}{3}=\frac{33}{4}\times\frac{3}{11}=2\cdot25$ *Ans.*

In Ex. (1), the last figure in the answer would have been 4, if we had simply added the figures to 5 places past the point. Similarly, the answer to (2) would have been ·66317. If the figure following the last one retained be 5 or more than 5, then the accuracy will be still greater if the last figure retained be increased by 1. The answer to the first example is more nearly 9·90026 than 9·90025.

Ex. 38.

Find the value (correct to 5 decimal places) of

1. $37\cdot\dot{4}\dot{9}\dot{2} + 5\cdot6\dot{3}\dot{1} + \cdot7\dot{0}0\dot{4} + 29\cdot\dot{3}5\dot{9}\dot{7} + \cdot\dot{8} + 7\cdot5\dot{4}\dot{4}$.
2. $\cdot00\dot{9}\dot{8} + 7\cdot\dot{7}\dot{8}\dot{6}\dot{9} + 27\cdot73 + 6\cdot\dot{9} + \cdot\dot{8}5\dot{4}\dot{7} + \cdot0\dot{9}\dot{6}$.
3. $45\cdot3\dot{7}\dot{6} + \cdot7\dot{9}\dot{4} + \cdot\dot{0}0\dot{7} + 86 + 73\cdot\dot{7}\dot{9}\dot{4}\dot{7}\dot{3} + \cdot0\dot{6}$.
4. $7\cdot6\dot{6}5\dot{5} + \cdot0\dot{7}00\dot{3} + \cdot964 + 23\cdot7 + 6\cdot\dot{8}14\dot{9}\dot{7}\dot{6} + \cdot\dot{9}\dot{8}$.
5. $4\cdot6\dot{1}\dot{3} - \cdot8\dot{1}\dot{7}$.
6. $\cdot\dot{3}\dot{1}\dot{4} - 2956$.
7. $62\cdot01 - 58\cdot0\dot{7}$.
8. $\cdot37 - \cdot\dot{3}\dot{6}$.
9. $15\cdot\dot{0}0\dot{8} - \cdot\dot{8}00$.
10. $2\cdot031 - 1\cdot\dot{7}\dot{6}\dot{4}$.

Find the value of

11. $\cdot\dot{8}1\dot{4} \times \cdot\dot{4}\dot{5}$.
12. $\cdot\dot{1}42857 \times \cdot\dot{6}$.
13. $3\cdot125 \times \cdot\dot{4}\dot{6}$.
14. $3\cdot\dot{0}\dot{2} \times \cdot\dot{6}\dot{4}$.
15. $3\cdot\dot{2}1\dot{4}2857 \times \cdot\dot{3}\dot{8}$.
16. $\cdot\dot{8}\dot{4} + \cdot795\dot{4}$.
17. $1\cdot\dot{3}\dot{1}42857 + 2\cdot76$.
18. $3\cdot625 + 9\cdot\dot{6}$.
19. $(\cdot25 + \cdot41\dot{6}) + \cdot\dot{6}$.
20. $(1\cdot5 + \cdot\dot{8}\dot{3} - \dot{7}) + 3\cdot\dot{1}$.

Second Course.

1. Express $2\frac{1}{2} + 3\frac{1}{4} - 4\frac{1}{8} + 5\frac{1}{8} - 6\frac{1}{8}$ as a decimal.
2. Add together the sum, difference, and product, of $5\frac{1}{7}$ and $4\frac{3}{8}$, and express the result as a decimal.
3. Divide $4\frac{1}{20}$ by $\cdot3\dot{4}\dot{8}$, and multiply the result by $\frac{11}{16}\frac{5}{7}\frac{5}{5}$.
4. How much must be added to 5·6 to make it $5\cdot\dot{6}$?
5. By what decimal must $\cdot41\dot{6}$ be multiplied to make it 1?
6. One-third of a ship belongs to A, $\cdot\dot{3}\dot{8}$ to B, and the remainder to C. What decimal represents C's share?
7. Divide ·0017907 by ·00564, and multiply the quotient by $3\frac{1}{1}\frac{8}{3}\frac{9}{7}$.
8. Express $(3\frac{1}{4} - 2\frac{1}{3} + \frac{3}{6}) \div (2\frac{3}{4}$ of $\frac{5}{2\frac{1}{2}} - \frac{7}{1\frac{1}{2}})$ as a decimal.
9. $\dfrac{\cdot054 \text{ of } 2\cdot8}{\cdot00126}$.
10. $\dfrac{(2\frac{1}{2} + 3\frac{1}{3}) \div \cdot61\dot{6}}{\cdot0027}$.
11. $\dfrac{4\frac{1}{2} \text{ of } 2\frac{5}{12} \div 1\cdot2083}{3\cdot0125 \times \cdot4\dot{9}}$.
12. $\dfrac{1\frac{1}{2} + 2\frac{1}{2} - 3\frac{1}{3} + 4\frac{1}{8} - 1\frac{1}{10}}{2\cdot5 \times \cdot001 \times \cdot00\dot{1}}$.

13. Find the value of $\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\frac{1}{720}+\frac{1}{5040}+\frac{1}{45360}$ correct to 6 places of decimals.

14. Two-fifths of the population in a parish are between 15 and 30 years of age; $\cdot\dot{4}28571\dot{4}$ of the population are under 15 years; the remaining 2730 are above 30 years. What is the population of the parish?

15. Find the sum of the figures in the circulating decimal equivalent to $\frac{1}{17}$.

16. An express train travels 129 miles in $3\cdot58\dot{3}$ hours. What is the rate per hour?

17. The quotient in a division is $5\frac{8}{11}$, and the divisor is $1\cdot3\dot{1}\dot{8}$; find the dividend.

18. By how much does the product of 7·625 and $\frac{44}{61}$ exceed the quotient of $9\frac{3}{8}$ by $2\cdot\dot{8}46153\dot{8}$?

19. The pendulum of a clock makes $302\cdot\dot{2}$ vibrations in 5 minutes instead of beating seconds' time. How much too fast will the clock be in 24 hours?

20. Standard silver is composed of ·925 parts of pure silver, and the remainder copper; and a pound Troy of standard silver is coined into 66 shillings. Express, as the decimal of an ounce, the weight of pure silver in 5 shillings.

DECIMAL COINAGE.

64. The operations of addition, subtraction, multiplication, and division, in connexion with the proposed decimal coinage are the same as simple addition, subtraction, &c.

Ex. 1. Add together £3. 2fl. 7c. 5m.; 8fl. 3m.; £6. 2c.; £9. 4m.
Ex. 2. From £13. 4fl. 3c. take £7. 6fl. 5c. 2m.
Ex. 3. Multiply £3. 2fl. 5m. by 24.
Ex. 4. Divide £165. 2fl. 4m. by 234.

(1).	(2).	(3).	(4).
3275	13430	3205	234)165204(706
803	7652	24	1638
6020	5778	12820	1404
9004		6410	1404
19102	*Ans.*, £5. 7fl. 7c. 8m.	76920	*Ans.*, 7fl. 6m.
Ans., £19. 1fl. 2m.		*Ans.*, £76. 9fl. 2c.	

It should be noticed that when any denomination (except pounds) is omitted, a 0 is written in the place of that denomination. Also, if the question involves the lowest denomination (mils), then in the answer the right hand figure is mils, the next cents, the next florins, and all the rest pounds.

Ex. 39.

Find the sum of—

1. £3. 4fl. 2c. 3m.; £6. 7fl. 4c. 1m.; £29. 3fl. 9c. 5m.; £6. 1fl. 2c. 4m.
2. £18. 7c.; 5fl. 9m.; £6. 3fl.; £7. 8m.; £194. 5c.
3. £235. 6c. 8m.; £9. 8fl. 4m.; £17.6m.; 8fl. 2m.

Subtract—

4. £8. 5c. 6m. from £23. 8fl. 2m.
5. £13. 2c. from £180. 9m.
6. 9fl. 8c. 7m. from £3. 4fl.
7. £20. 3m. from £27. 8c.
8. £560. 9c. from £700. 1fl. 2m.
9. £9. 9fl. 9c. 9m. from £10.

Multiply—

10. £3. 2fl. 1c. 5m. by 7.
11. £6. 8c. 7m. by 26.
12. 7fl. 4c. 5m. by 40.
13. £9. 6c. by 75.
14. £3. 8fl. 4m. by 265.
15. £9. 3fl. 7c. 5m. by 288.

Divide—

16. £7. 8fl. 4c. 2m. by 6.
17. £3. 4fl. 3m. by 9.
18. £1. 2c. 4m. by 64.
19. £26648. 4fl. by 848.
20. Take half-a-guinea from 7fl. 3c., and multiply the remainder by 216.

REDUCTION OF FRACTIONS.

65. *To find a fractional part of any concrete number.*

RULE. Multiply the number by the numerator of the fraction, and divide the product by the denominator, if there be a remainder, proceed to each successive lower denomination as in compound division.

Ex. 1. Find the value of $\frac{4}{5}$ of £2.

$$\frac{4}{5} \text{ of } £2. = \frac{4 \times 40s.}{5} = 32s. = £1.\ 12s., \textit{Ans.}$$

Ex. 2. Find the value of $2\frac{3}{7}$ of 15s. 9d.

$$2\frac{3}{7} \text{ of } 15\text{s. } 9\text{d.} = \frac{16 \times 189\text{d.}}{7} = 432\text{d.} = 36\text{s.} = £1.\ 16\text{s.}, \textit{Ans.}$$

Ex. 3. Find the value of $\frac{3}{4}$ of $\frac{5}{6}$ of a ton.

$$\frac{3}{4} \text{ of } \frac{5}{6} \text{ of 1 ton} = \frac{5}{8} \text{ of 1 ton} = \frac{5 \times 20}{8} \text{ cwt.} = \frac{25}{2} \text{ cwt.} = 12\text{cwt. } 2\text{qrs.}, \textit{Ans}$$

Ex. 4. Find the value of $\frac{3}{8}$ of 1s. + $\frac{5}{6}$ of £1 + $\frac{5}{7}$ of $\frac{4}{5}$ of 1 guinea.

$$\frac{3}{8} \text{ of } 1\text{s.} = \frac{3 \times 12}{8}\text{d.} = \frac{9}{2}\text{d.} = \quad 4\tfrac{1}{2}\text{d.}$$

$$\frac{5}{6} \text{ of } £1. = \frac{5 \times 20}{6}\text{s.} = \frac{50}{3}\text{s.} = \quad 16\text{s. } 8\text{d.}$$

$$\frac{5}{7} \text{ of } \frac{4}{5} \text{ of 1 gui.} = \frac{4}{7} \text{ of 1 gui.} = \frac{4 \times 21}{7} = \quad 12\text{s. } 0\text{d.}$$

$$£1.\ 9\text{s. } 0\tfrac{1}{2}\ \textit{Ans.}$$

Ex. 40.

Find the value of—

1. $\frac{3}{5}$ of 5s.; $\frac{1}{4}$ of 4s. 8d.; $\frac{5}{6}$ of 7s. 6d.; $\frac{2}{9}$ of 5s. 3d.

2. $\frac{4}{11}$ of 17s. 5d.; $\frac{2}{3}$ of £1. 13s.; $\frac{3}{7}$ of £8. 1s.; $\frac{2}{5}$ of 3s. $5\frac{1}{2}$d.

3. $\frac{2}{11}$ of 8s. $8\frac{1}{2}$d. $\frac{7}{8}$ of £3.; $\frac{12}{13}$ of 7s. $10\frac{1}{2}$d.; $\frac{5}{16}$ of £3. 5s. 4d.

4. $\frac{2}{3}$ of $\frac{3}{4}$ of 10s.; $\frac{5}{6}$ of $\frac{9}{10}$ of 12s. 4d.; $\frac{1}{2}$ of $\frac{3}{5}$ of £4. 17s. 1d.

5. $\frac{7}{11}$ of 1 mile; $\frac{5}{7}$ of 1 cwt.; $3\frac{1}{3}$ of 1s. 8d.; $\frac{4}{15}$ of £5. 10s. $11\frac{1}{4}$.

6. $\frac{3}{7}$ of $\frac{5}{6}$ of $\frac{5}{64}$ of a ton; $\frac{4}{11}$ of $\frac{3}{14}$ of $2\frac{1}{4}$ of 3 hogsheads.

7. $3\frac{1}{8}$ of $\frac{7}{15}$ of $2\frac{1}{7}$ of 3 days 5 hrs.; $2\frac{1}{12}$ of $3\frac{3}{7}$ of 15s. 9d.

8. $\frac{3}{4}$ of a guinea + $\frac{5}{6}$ of a crown + $\frac{3}{11}$ of $\frac{1}{8}$ of $12\frac{1}{7}$ of £13. 17s. 1d.

9. $\frac{9}{11}$ of $3\frac{1}{3}$ of $\frac{5}{18}$ of 7s. 8d. + $\frac{3}{7}$ of $\frac{5}{9}$ of £1. 2fl. 6c. + $\frac{3}{28}$ of 1s. $5\frac{1}{2}$d.

10. From $2\frac{1}{2}$ of $5\frac{1}{3}$ of 3 acres take $3\frac{1}{4}$ of $\frac{2}{11}$ of 5 acres.

11. Express in Troy weight, $\frac{3}{14}$ of 5lbs. Avoir. $+\frac{7}{10}$ of 3drs. 2scr. $+\frac{5}{24}$ of 2lbs. Troy + 1000 grains.

12. How many gallons are there in $\frac{5}{13}$ of $4\frac{1}{4}$ of $2\frac{1}{15}$ of $1\frac{3}{17}$ of 3 bushels?

66. *To reduce one concrete number to the fraction of another of the same kind.*

RULE. Write the number to be reduced as the numerator of a fraction, and the number to which it is to be reduced as the denominator; reduce both numbers to the same denomination, and finally reduce the fraction to its lowest terms.

Ex. 1. Reduce 16s. to the fraction of £1.

$$\frac{16\text{s.}}{£1.} = \frac{16\text{s.}}{20\text{s.}} = \frac{4}{5} \textit{ Ans.}$$

Ex. 2. Reduce $\frac{3}{4}$ of $\frac{3}{5}$ of 7s. 6d. to the fraction of $\frac{5}{6}$ of $\frac{3}{10}$ of 18s.

$$\frac{\frac{3}{4} \text{ of } \frac{3}{5} \text{ of } 7\text{s. } 6\text{d.}}{\frac{5}{6} \text{ of } \frac{3}{10} \text{ of } 18\text{s.}} = \frac{\frac{3}{4} \text{ of } \frac{3}{5} \text{ of } 15 \text{ sixpences}}{\frac{5}{6} \text{ of } \frac{3}{10} \text{ of } 36 \text{ sixpences}}$$

$$= \frac{\overset{1}{\cancel{3}}. \overset{2}{\cancel{3}}. \overset{1}{\cancel{3}}. \overset{1}{\cancel{10}}. \overset{3}{\cancel{15}}}{\underset{1}{\cancel{4}}. \underset{1}{9}. \underset{1}{\cancel{5}}. \underset{1}{\cancel{3}}. \underset{3}{\cancel{36}}} = \frac{9}{17} = 1\frac{1}{17} \textit{ Ans.}$$

Ex. 41.

Reduce—

1. 2s. 6d. to the fraction of £1; $1\frac{1}{2}$d. to the fraction of 1s.

2. 3s. 4d.; 7s. 6d.; 13s. 4d.; 12s. 6d.; each to the fraction of £1.

3. 3s. $4\frac{1}{2}$d. to the fraction of 10s. $1\frac{1}{2}$d.; $8\frac{1}{2}$d. to the fraction of 5s. 8d.

4. £1. 4s. 8d. to the fraction of 16s.; 17s. 6d. to the fraction of £3. 5s.

5. 3s. 6d. to the fraction of a guinea; half-a-guinea to the fraction of a moidore; 11s. 7d. to the fraction of £4. 1fl. 7c.

6. 3qrs. 7lbs. to the fraction of 9cwt. 3qrs.; $3\frac{1}{2}$ furlongs to the fraction of 1 mile; 5yds. 1qr. 1na. to the fraction of 17 Eng. ells.

7. $\frac{3}{4}$ of £1 to the fraction of $2\frac{1}{2}$ guineas; $\frac{5}{8}$ of $1\frac{3}{4}$ of 4s. to the fraction of $1\frac{1}{8}$ of $\frac{1}{5}$ of a crown.

8. What part of a bushel is $\frac{4}{5}$ of $2\frac{3}{5}$ of $1\frac{1}{4}$ of 3 quarts?

9. What part of a pound Avoir. is $\frac{1}{8}$ of $\frac{5}{13}$ of a pound Troy?

10. A man earns £1. 9s. a-week: express the daily earnings of three men at the same rate as the fraction of £1.

11. If a quarter of wheat weighs 300lbs. what part of a *cental* is a peck?

12. Express $\frac{3}{4}$ of a crown + $\frac{5}{6}$ of a guinea as the fraction of £1. 7fl. 8c. 5m.

67. *To find the value of any decimal of a concrete number.*

RULE. Multiply the decimal by the number required to reduce the concrete quantity to its next lower denomination, and mark off in the product as many figures to the right of the point as there are in the decimal itself: if necessary, proceed in the same way to each successive lower denomination. The figures to the *left* of the points represent the different parts of the concrete number in each successive denomination.

Ex. (1). Find the value of ·875 of £1.; (2). ·575 of 3 tons.

·875	·575
20	3
17·500 shillings	1·725 tons
12	20
6·000 pence	14·500 cwt.
	4
	2·000 qrs.

Ans., 17s. 6d. *Ans.*, 1 ton 14 cwt. 2 qrs.

(3). Find the value of 4·32 of £2. 16s. 3d.; (4). ·655 of an acre.

£2. 16s. 3d. = 675d.

4·32	·655
6 75	4
21 60	2·620 roods
302 4	40
2592	24·800 poles
12)2916·00 pence	$30\frac{1}{4}$
20)243 shillings	24000
£12. 3s., *Ans.*	200
	24·200 yards

Ans., 2 ro. 24 po. 24·2 yds.

Circulating decimals must be reduced to vulgar fractions.

Ex. 5. Find the value of $\cdot\dot{2}\dot{7}$ of 5s. 6d.

$$\cdot\dot{2}\dot{7} \text{ of } 5\text{s. } 6\text{d.} = \tfrac{3}{11} \text{ of } 66\text{d.} = \frac{27 \times 66\text{d.}}{99} = 18\text{d.} = 1\text{s. } 6\text{d., } \textit{Ans.}$$

It should be noticed that a near approximation to the answer may generally be obtained in the usual way, without reducing the circulator to a vulgar fraction. Thus, in Ex. 5, $\cdot\dot{2}\dot{7} \times 66\text{d.} = 17{\cdot}82\text{d.} = 1\text{s. } 5\text{d. } 3{\cdot}28\text{f.}$

Ex. 42.

Find the value of—

1. ·25 of £1.; ·75 of 1s.; ·125 of £3.; ·625 of 12s.
2. ·0625 of 5s. 4d.; 2·75 of £3. 4s.; ·0078125 of £1. 1s. 4d.
3. 4·128 of £1. 0s. 10d.; ·015625 of 8 guineas; 1·875 of a crown.
4. 7·0625 of a cwt.; ·2725 of 3 tons; ·0075 of 1 lb. Troy.
5. 3·24 of 5 acres; ·035 of a day; $\cdot\dot{8}\dot{6}$ of 2 miles.
6. ·375 of £1. + 3·4 of 1s. 8d. + $\cdot 4285\dot{7}\dot{1}$ of a guinea.
7. $4{\cdot}\dot{4}$ of 11s. 3d. + $\cdot\dot{4}\dot{6}$ of 3s. 9d. + $\cdot 1\dot{8}\dot{5}$ of a moidore.
8. $\cdot\dot{5}$ of an Eng. ell; $\cdot 0769\dot{2}\dot{3}$ of 1s. $10\frac{3}{4}$d.; $\cdot\dot{7}\dot{7}$ of 9 florins.
9. Multiply £4. 16s. 6d. by 3·375; divide £1758. 9s. $9\frac{3}{4}$d. by $712{\cdot}\dot{3}$.
10. From ·847 of a crown subtract ·035 of a guinea.
11. If a pound of tea cost $3{\cdot}7\dot{2}$s. what is the value of a chest containing 54 lbs?
12. A gentleman's gross income is £1500; what is his net income after deducting an income tax of $6{\cdot}\dot{5}$ per cent.?

68. *To reduce one concrete number to the decimal of another of the same kind.*

Rule. (1). Express both numbers in the same denomination, and divide that which it is required to reduce by the other, as in division of decimals: or (2), form a fraction with the numbers and reduce it to a decimal by (59).

Ex. (1). Reduce 19s. to the decimal of £4.; (2). 4s. 7d. to the decimal of 5s. 4d.

(1). £4. = 80s.

80)19·0000(·2375 *Ans.*
160
300
240
600
560
400
400

(2). 4s. 7d. = 55d.
5s. 4d. = 64d.

64 { 8)55·000000
8) 6·875000
·859375 *Ans.*

Ex. 3. Reduce 11¼d. to the decimal of 1s. 3¾d.

11¼d. = 45f.; and 1s. 3¾d. = 63f.;

∴ fraction required is $\frac{45}{63}=\frac{5}{7}=\cdot\dot{7}14285\dot{}$ *Ans.*

When the given number is to be reduced to the decimal of another consisting of a *single* denomination, we may proceed as follows:—

Ex. 4. Reduce 17s. 6¾d. to the decimal of £1.

4| 3·00 f.
12| 6·7500 d.
20|17·56250 s.
·878125 *Ans.*

Here we first reduce ¾d. which = ·75d. Then prefixing the 6d. we have 6·75d. Next, dividing by 12 we have ·5625s. Then, prefixing the 17s. we have 17·5625s.; which divided by 20 gives ·878125.

If it had been required to reduce 17s. 6¾d. to the decimal of £5. instead of £1., we should have had to divide the last quotient ·878125 by 5. If it had been 7s. instead of £1., we should have divided 17·5625 by 7.

Ex. 43.

Reduce—

1. 7s. to the decimal of £1.; and 15s. to the decimal of £3.
2. 8d. to the decimal of 3s. 4d.; and 7½d. to the decimal of 1s.
3. 2s. 11d. to the decimal of 4s. 8d.; and £3. 15s. to the decimal of £2. 8s.
4. 2cwt. 2qrs. to the decimal of a ton; and 3 furlongs to the decimal of a mile.
5. An ounce Troy to the decimal of an oz. Avoir.; and 9½d. to the decimal of 1s. 8d.
6. ⅘ of 2s. 6d. to the decimal of ⅝ of 6s.; and 13s. 6¼d. to the decimal of £6. 4fl. 9c.
7. What decimal of a week is 16hrs. 48 minutes?
8. What decimal of a guinea is £·04746?
9. A garden contains 847 sq. yds.; what decimal of an acre is it?

10. The population of a village is 448, and 203 do not exceed 20 yrs. of age; what decimal of the whole are above this age?

11. Express 7fl. 3m. as the decimal of half-a-guinea.

12. A school contains 198 scholars, but 27 are absent: express the number of absentees out of 100 scholars as a decimal.

Second Course.

1. Find the value of $\frac{3}{8}$ of $1\frac{4}{11}$ of £1. 13s., and express the result as the decimal of £12.

2. The log of a ship's course from Liverpool to Melbourne gives 19950 miles, and the voyage is made in 95 days. Express the average rate of sailing per hour as the decimal of a mile.

3. A's capital is $\cdot\dot{7}\dot{2}$ of B's; they enter into partnership and their profits at the end of the year amount to £217. What is B's share?

4. A chest of tea containing 54lbs. is bought for £8. 5s. and sold at $4\cdot2\dot{7}$s. per lb. What is the profit on the whole?

5. Reduce 3·85s. to the decimal of 4·4 of a guinea.

6. What fraction of 5s. is equivalent to $\cdot1\dot{6}$ of a pound?

7. A's money is ·625 of B's, but if A had 1s. 6d. more his money would be $\frac{3}{4}$ of B's. How much has each?

8. Multiply $\frac{1}{7}$ of half-a-guinea by $\cdot0\dot{1}4285\dot{7}$.

9. A pound of coffee is worth $3\cdot6\dot{1}$lbs. of sugar; a pound of tea is worth $2\cdot\ddot{2}\dot{7}\ddot{1}$lbs. of coffee; and a pound of sugar costs 6d. What is the price of the tea per lb.?

10. Express 7fl. 6c. 9m. as the decimal of £2. 8s. $0\frac{3}{4}$d.

11. A gentleman after paying an income tax of $\cdot58\dot{3}$s. in the pound has £435. 12s. 6d. remaining: find his income.

12. Find the difference between $2\cdot\dot{7}$ of $\cdot\dot{6}\dot{1}$ of $\cdot4\dot{3}\dot{6}$ of £5. and $\frac{1}{8}$ of $4\frac{1}{4}$ of $\frac{55}{100}$ of £3. 17s.

13. Divide 2 tons 3cwts. 3qrs. 12lbs. by $7\cdot3\dot{7}\dot{5}$.

14. A fifty-gallon cask is filled with 94·75 per cent. of wine and the remainder brandy; how many bottles of brandy does the cask contain, supposing a bottle to consist of $1\frac{1}{2}$ pints?

15. Three boats start together on a mile race; the first gains 3·2yds. in every 100 on the second, and the second gains 2·7yds. in every 145·2yds. on the third. Find the distance between the first and third boats at the end of the race?

16. In French money 100 centimes make one franc: find the value of £3. 8s. in French money, 25·25 francs being given in exchange for one pound sterling.

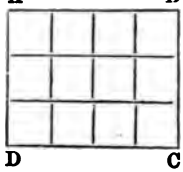
SQUARE AND CUBIC MEASURE.

69. A *Rectangle* is a surface contained by four straight lines, all its angles being right angles.

A *Rectangular Solid* is one whose faces are all rectangles.

A boy's slate and the page of a book are rectangles. A common building brick is a rectangular solid.

The annexed figure represents a rectangle. If we suppose the length A B to be 4 feet, and the breadth A D to be 3 feet, then, by dividing A B into 4 equal parts, and A D into 3 equal parts, and drawing lines across the figure we divide the whole surface into 12 equal parts, each being a foot square. Also by multiplying the length and breadth together we have $4 \times 3 = 12$. Hence the followlng rule—



To find the area of a rectangle.

RULE. Multiply the length by the breadth.

Again, if we suppose the figure to represent the top of a block of wood 4ft. long, 3ft. broad, and 2ft. thick, it is evident that it might be divided into two equal blocks, each being 1ft. thick. Each block would then contain 12 cubic feet, and consequently the two together would contain 24 cubic feet. Also by multiplying the length, breadth, and thickness together, we have $4 \times 3 \times 2 = 24$. Hence the following rule—

To find the content of a rectangular solid.

RULE. Multiply the length, breadth, and thickness together.

It should be carefully noticed that in such expressions as "multiplying the length by the breadth," it is the abstract *numbers* that are multiplied and not *quantities*, feet, inches, &c.

Ex. 1. The top of a table is 4ft. 6in. long, and 3ft. 4in. broad: how many square feet does it contain?

1st method.

4ft. 6in. = 54in; 3ft. 4in. = 40in.

$$\begin{array}{r} 54 \\ 40 \\ \hline 144)2160(15 \\ 144 \\ \hline 720 \\ 720 \\ \hline \end{array}$$

15sq. ft., *Ans.*

2nd method.

$$4\tfrac{1}{2} \times 3\tfrac{1}{3} = \tfrac{9}{2} \times \tfrac{10}{3} = \tfrac{90}{6} = 15$$

15sq. ft., *Ans.*

In the second method we express 4ft. 6in. and 3ft. 4in. as $4\frac{1}{2}$ft. and $3\frac{1}{3}$ft. respectively, and proceed as in multiplication of fractions.

Ex. 2. A block of marble is 6ft. 9in. long, 5ft. 4in. broad, and 4ft. 6in. thick; how many cubic yards does it contain?

6ft. 9in. $= 6\frac{3}{4}$ft.; 5ft. 4in. $= 5\frac{1}{3}$ft.; 4ft. 6in. $= 4\frac{1}{2}$ft.

whence, $6\frac{3}{4} \times 5\frac{1}{3} \times 4\frac{1}{2} = \frac{27.\ 16.\ 9}{4.\ 3.\ 2}$

$= 162$ cub. ft. $= 6$ cub. yds., *Ans.*

We might have reduced the dimensions to inches and then divided the product by 1728 and by 27.

Ex. 3. How much will it cost to cover the floor of a room 22ft. long and 18ft. broad, with carpet at 3s. 6d. per sq. yd.?

$$\frac{22 \times \overset{2}{\cancel{18}} \times 7}{\underset{1}{\cancel{9}} \times \underset{1}{\cancel{2}}} = 154\text{s.} = £7.\ 14\text{s.}, \textit{Ans.}$$

Here we divide by 9 to convert the sq. ft. into yds., and express 3s. 6d. as $\frac{7}{2}$s.

Ex. 4. What will be the expense of papering the walls of a room 22ft. long, 14ft. broad, and 12ft. high, with paper $\frac{3}{4}$ of a yard wide, at $4\frac{1}{2}$d. per yd.?

The two side walls are two equal rectangles, and the two end walls are also two equal rectangles. If then we conceive the four walls to be *opened out* they would form one rectangle, whose breadth would be the height of the room, and whose length would be twice the length of the room added to twice its breadth. Hence, to find the area, we add the length and breadth of the room together, double it, and multiply by the height.

$\frac{(22+14) \times 2 \times 12}{9}$ = area of walls in sq. yds.

$\frac{36\ .\ 2\ .\ 12}{9 \times \frac{3}{4}}$ = number of yds. of paper required.

$\frac{36\ .\ 2\ .\ 12\ .\ 4\frac{1}{2}}{9 \times \frac{3}{4}}$ = cost of paper in pence.

$\frac{36\ .\ 2\ .\ 12\ .\ 4\frac{1}{2}}{9 \times \frac{3}{4} \times 12}$ = cost of paper in shillings.

$\frac{36\ .\ 2\ .\ 12\ .\ 4\frac{1}{2}}{9\ .\ \frac{3}{4}\ .\ 12\ .\ 20}$ = cost of paper in pounds.

$$= \frac{\overset{12}{\cancel{36}}\ .\ \overset{2}{\cancel{2}}\ .\ \overset{1}{\cancel{4}}\ .\ \overset{1}{\cancel{12}}\ .\ \overset{3}{\cancel{9}}}{\underset{1}{\cancel{3}}\ .\ \underset{1}{\cancel{3}}\ .\ \underset{1}{\cancel{12}}\ .\ \underset{5}{\cancel{20}}\ .\ \underset{1}{\cancel{2}}} = £\frac{12}{5} = £2.\ 8\text{s.}, \textit{Ans.}$$

Ex. 5. What will be the expense of papering a room 28ft. long, 17ft. wide, and 15ft. high, with paper $\frac{5}{8}$ of a yard wide, at 5½d. per yd. ?

$$\frac{(28+17)\times 2\times 15\times 5\frac{1}{2}}{9\times\frac{5}{8}\times 12\times 20}=\frac{45\,.\,2\,.\,15\,.\,21\,.\,8}{9\,.\,5\,.\,12\,.\,20\,.\,4}$$

$=£\frac{21}{4}=£5.\ 5s.$, *Ans.*

Ex. 6. How many bricks will be required in building a wall 22½ft. long, 9ft. high, and 13in. thick, the dimensions of a brick being 9in. long, 4½in. broad, and 2½in. thick ?

Here we are required to divide the cubical content of the wall by the content of a brick.

$$\frac{22\frac{1}{2}\times 12\times 9\times 12\times 13}{9\times 4\frac{1}{2}\times 2\frac{1}{2}}=\frac{45\,.\,12\,.\,9\,.\,2\,.\,4\,.\,12\,.\,13}{2\,.\,9\,.\,9\,.\,9}$$

$=4160$ number required.

In relation to the rectangle we have seen that—

$$\text{area} = \text{length} \times \text{breadth}.$$
$$\therefore \text{length} = \text{area} \div \text{breadth}.$$
$$\text{and, breadth} = \text{area} \div \text{length}.$$

Similarly, in the case of rectangular solids,—

$$\text{content} = \text{length} \times \text{breadth} \times \text{thickness}.$$
$$\therefore \text{length} = \frac{\text{content}}{\text{breadth} \times \text{thickness}}.$$

and similarly for the other dimensions.

Also, since the area of any one face of a rectangular solid is the product of the two dimensions of that face, it is evident that the cubical content divided by the area of any one face will give the remaining linear dimension of the solid.

Ex. 7. The floor of a room which is 16ft. wide contains 332sq. ft.; find the length of the room.

$\frac{3\,3\,2}{1\,6}=20\frac{1}{16}\text{ft.}=20\frac{3}{4}\text{ft.}=20\text{ft. }9\text{in.}$, *Ans.*

Ex. 8. A block of granite 12ft. long and 6ft. broad, is sold for £24., at 2s. 6d. per cub. ft.: find the thickness of the block.

$$\frac{24\times 20}{2\frac{1}{2}\times 12\times 6}=\frac{24\,.\,20\,.\,2}{5\,.\,12\,.\,6}=\frac{8}{3}\text{ft.}=2\text{ft. }8\text{in.}, \textit{Ans.}$$

Ex. 9. The length of the side of a square field is 242yds.: find its area.

$$\text{Area}=\frac{242\times 242}{4840}=\frac{121}{10}\text{ac.}=12\text{ac. }16\text{po.}$$

Ex. 10. A piece of deal 16ft. long is 11in. broad at one end and *7in. broad* at the other: how much will it cost at 2½d. per sq. ft. ?

Here we add the breadths of the two ends together, and divide by 2 to get the mean breadth.

$$\frac{(11+7)\times 16\times 12\times 2\frac{1}{2}}{2\times 144}=\frac{18\,.\,16\,.\,12\,.\,5}{2\,.\,144\,.\,2}=30\text{d.}=2\text{s. }6\text{d.}$$

In working questions relating to measurements, it is always desirable to draw the geometrical figure when it is not a simple rectangle. It is equally desirable that the teacher should be able to draw with tolerable accuracy on the black-board, the perspective representation of any rectilinear solid. A clearer notion will often be conveyed in this manner than it is possible to give by mere word-explanation.

Ex. 44.

1. A boy's slate is 14in. long and 10in. wide: how many sq. inches are there in its surface?
2. How many sq. feet are there in the floor of a room which is 16ft. 6in. long, and 14ft. wide?
3. A black-board is 3ft. 6in. long and 2ft. 8in. broad: how much did it cost at $7\frac{1}{2}$d. per sq. ft.?
4. Find the value of a plot of building ground 132yds. long and 55yds. wide, at 3s. 6d. per sq. yd.
5. What will be the expense of flooring a room 24ft. long and 16ft. wide at $3\frac{3}{4}$d. per square foot?
6. How much will it cost to paper the walls of a room 21ft. long, 18ft. wide, and 15ft. high, with paper at $6\frac{1}{2}$d. per sq. yd.?
7. How many yards of carpet $\frac{3}{4}$ yd. wide will be required for a room which measures 27ft. by 19ft.?
8. It requires 35sq. yds. of carpet to cover the floor of a room which is 15 wide: what is the length of the room?
9. How many cubic ft. are there in a piece of mahogany which is 13ft. 6in. long, 2ft. 8in. wide, and 1ft. 3in. thick?
10. A garden is 53yds. long, and 38yds. broad: what will be the expense of making a gravel walk round it 6ft. broad, at 1s. 3d. per sq. yd.?
11. What will be the expense of glazing 8 windows, each of which requires 12 panes 15in. long and 11in. wide, at $10\frac{1}{2}$d. per sq. ft.?
12. How many square feet are there in the surface of a cube whose edge measures 3ft. 6in.?
13. What will be the cost of papering the walls of a room 19ft. long, 17ft. wide, and 15ft. high, with paper $\frac{4}{5}$ of a yard wide, at $4\frac{1}{2}$d. per yard?
14. How many yards of carpet $\frac{5}{8}$ of a yard wide will be required for the floor of a room which measures 29ft. by 22ft. 6in.?
15. The front of a house is 42ft. wide and 39ft. high; what will be the expense of pointing the brickwork at 1s. 6d. per sq. yd., deducting the space occupied by 8 windows, each measuring 6ft. by 4ft., and a door which measures 8ft. by 4ft. 6in.?

16. A field is 343yds. long and 230yds. wide: find its area.

17. The length of a book is 8½in., breadth 7in., and thickness 2¼in.; how many copies will be required to fill a box, the inside dimensions of which are 3ft. 6in., 2ft. 10in., and 2ft. 3in.?

18. A picture frame is 3½in. broad and 1¾in. thick; the size of the picture inside the frame is 25in. by 17in.; how much will it cost to gild the front and sides of the frame, at ½d. per sq. in.?

19. The foundation for a house is dug out in the form of a rectangular trench, 22½yds. long and 19yds. wide: how many cubic yards of earth will be taken out, the trench being 2ft. 6in. wide and 4ft. deep?

20. If a cubic inch of iron weighs 3¼ oz., what will be the weight of an iron bar 14ft. long, 4in. broad, and 2in. thick?

21. How many square feet of ½-inch deal can be cut out of a beam 28ft. long and 16 inches square, allowing 3 inches waste for sawing?

22. How much will the flooring of a room 20ft. long and 16ft. wide cost, at £2. 5s. the square?

23. What will be the expense of building a brick wall 80yds. long and 9ft. high, at £12. 2s. per rod?

24. A garden 65yds. long and 38yds. wide, has a gravel walk 5ft. broad running all round it, and also lengthways down the middle, and across the middle: how many square yards are left for cultivation?

25. A brick is 9in. long, 4½in. broad, and 2½in. thick; how many square inches are there in its surface?

26. A cistern 6ft. 6in. long, 4ft. 6in. wide, and 3ft. 6in. deep, is to be lined with lead weighing 5lbs. per square foot: what will be the expense, the lead being 4d. per lb.?

27. What will be the expense of papering a room 24ft. 6in. long, 17ft. 6in. wide, and 13ft. high, with paper ⅓ of a yard wide at 6½d. per yard, allowance being made for a cornice round the top of the wall 5in. deep, and a skirting round the bottom 7in. deep?

28. How many square feet are there in the surface of a garden roller 3ft. 6in. long, and 18in. diameter, the circumference of a circle being 3⅐ times its diameter?

29. How many square inches are there in the interior surface of a hollow cube, which will just contain a ball whose circumference is 6⅔ inches?

30. If a cubic foot of cast iron weigh 4½cwt., what will be the weight of a hollow cast iron pillar 15ft. long, and 9in. square, the hollow in the pillar being 3in. square?

31. A field 550yds. long and containing 28ac. 3ro., is to have four drains cut across its breadth; what will be the expense at 5d. per yd.?

32. If, in the last question, each drain is to be 2ft. wide at the top, 1ft. 4in. wide at the bottom, and 2ft. 6in. deep, how many cubic yards of earth will be dug out of the four drains?

70. *Cross Multiplication* or *Duodecimals* is another method of calculating measurements in common use among practical men. It is probable, however, that in course of time it will be almost wholly superseded by the methods already given.

Ex. 1. A school map measures 4ft. 3in. by 3ft. 6in.; find the area of its surface.

Here we first multiply 4ft. 3in. by 3, which gives 12sq. ft. and $\frac{9}{12}$ of a square ft. We next multiply the top line by 6′, thus, 6 times 3 are 18, 12 is contained in 18 once and 6 over; put down 6 on the right hand, and *carry* 1: then, 6 times 4 are 24 and 1 makes 25, which divided by 12 gives 2 and 1 over, the 1 being placed under the 9, and 2 under the 12. The result is, 14sq. ft. + $\frac{10}{12}$ of a sq. foot + 6sq. in., or 14sq. ft. + 120sq. in. + 6sq. in., that is, 14sq. ft. 126sq. in. In practice the 10′ 6″ is reduced to square inches by multiplying 10 by 12, and adding 6 to the product. The result, 14ft. 10′ 6″ is read 14 *square feet*, 10 *primes*, 6 *seconds*.

4ft. 3′	
3 . 6′	
12 . 9′	
2 . 1 . 6″	
14 . 10′ . 6″	*Ans.*, 14sq. ft. 126in.

Ex. 2. The edge of a cube measures 2ft. 4in.; find its content.

2ft. 4′	Reduction to cubic inches of
2 . 4′	8′ . 5″ . 4‴
4 . 8′	12
9′ . 4″	101
5 . 5′ . 4″	12
2 . 4′	1216 cub. in.
10 . 10′ . 8″	
1 . 9′ . 9″ . 4‴	*Ans.*, $(12 + \frac{8}{12} + \frac{8}{144} + \frac{4}{1728})$ cub. ft.
12 . 8′ . 5″ . 4‴	= 12 cub. ft. 1216 in.

Ex. 45.

Find the area of—

1. A board measuring 3ft. 6in. by 2ft. 8in.
2. A square map the side of which is 4ft. 4in.
3. The top of a table 6ft. 2in. long, and 4ft. 8in. broad.
4. Six windows, each 7½ft. by 3ft. 5in.
5. The surface of a cube whose edge measures 5½ft.
6. The surface of a block of stone 5ft 8in. long, 4ft. 9in. broad, and 2ft. 5in. thick.
7. The floor of a room 20ft. 3in. by 14ft. 7in.
8. A piece of deal 10ft. 6in. long is 1ft. 7in. broad at one end, and 13in. broad at the other: how many square feet does it contain?

9. Find the solid content of a cube whose edge measures 2ft. 7in.

10. What would be the expense of painting the surface of the cube in the last question at 6d. per square foot?

11. Two boards, each 5ft. 4in. long and 1ft. 4in. broad, are put together in the form of a cross: what will be the expense of painting the two faces of the cross at 2½d. per square foot?

12. What is the value of a block of stone whose length is 7ft. 6in., breadth 3ft. 4in., and thickness 1ft. 6in., at 1s. 6d. per cubic foot?

SIMPLE PROPORTION.

71. The *Ratio* of one number to another is the value of the former as measured by the latter.

The ratio of 12 to 20 is written 12 : 20, the first term 12 being called the *antecedent*, and 20 the *consequent*.

The value of a ratio is found by dividing the antecedent by the consequent; thus, $12 : 20 = \frac{12}{20} = \frac{3}{5}$. Hence the ratio 12 : 20 signifies that 12 is *three-fifths* of 20.

When concrete numbers form a ratio they must be of the same kind, for there can be no relation either of equality or inequality between *different* quantities.

A *Simple Proportion* is the equality of two ratios.

The ratios 9 : 15 and 12 : 20, being each equal to $\frac{3}{5}$, are equal to each other, and therefore form a proportion, which is written, 9 : 15 :: 12 : 20; and read, 9 *is to* 15 *as* 12 *is to* 20, signifying that 9 is the same part of 15 as 12 is of 20. Further, 9 and 20 are called the *extremes*, and 15 and 12 are called the *means*. All the terms are called *proportionals*, 9 being a *first* proportional to 15, 12, and 20; 15 being a *second* proportional to 9, 12, and 20; &c.

72. *The product of the extremes = the product of the means.*

For 9 : 15 :: 12 : 20, that is $\frac{9}{15} = \frac{12}{20}$, and multiplying each fraction by 15 × 20 we have $9 \times 20 = 12 \times 15$.

73. To find a first, second, third, or fourth proportional to any three numbers.

Let 4, 6, and 8, be the three given numbers; and let the letter x represent the proportional to be found; then,—

(1). $x : 4 :: 6 : 8$; $\therefore$ by (72), $8x = 24$, and $x = 3$.
(2). $4 : x :: 6 : 8$; ,, $6x = 32$, and $x = 5\frac{1}{3}$.
(3). $4 : 6 :: x : 8$; ,, $6x = 32$, and $x = 5\frac{1}{3}$.
(4). $4 : 6 :: 8 : x$; ,, $4x = 48$, and $x = 12$.

Hence, if any three terms in a proportion are given, the remaining term may be found; for,

$$\text{1st term} = \frac{\text{2nd} \times \text{3rd}}{\text{4th}}; \quad \text{2nd term} = \frac{\text{1st} \times \text{4th}}{\text{3rd}};$$

$$\text{3rd term} = \frac{\text{1st} \times \text{4th}}{\text{2nd}}; \quad \text{4th term} = \frac{\text{2nd} \times \text{3rd}}{\text{1st}}.$$

Ex. 46.

Find a first, second, third, and fourth proportional to—

1. 2, 4, 8.	2. 3, 6, 9.	3. 1, 2, 3.
4. 3, 4, 7.	5. 4, 1, 12.	6. 8, 6, 18.
7. 12, 16, 18.	8. 15, 24, 25.	9. 7, 11, 13.
10. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.	11. $2\frac{1}{2}$, $\frac{5}{6}$, $3\frac{1}{4}$.	12. ·02, $\frac{2}{5}$, ·008.

74. The *Rule of Three* is the application of the principles of proportion to concrete numbers.

Ex. 1. If 9lbs. of tea cost £1. 19s., what is the cost of 15lbs.?

This question may be treated in four different ways;—

(1). It is evident that the cost of 15lbs. will bear the same ratio to the cost of 9lbs. as 9lbs. does to 15lbs.; thus,—

cost of 15lbs. : £1. 19s. :: 15lbs. : 9lbs.

(2). The cost of 9lbs. will bear the same ratio to the cost of 15lbs. as 9lbs. bears to 15lbs.; thus,—

£1. 19s. : cost of 15lbs. :: 9lbs. : 15lbs.

(3). 15lbs. will bear the same ratio to 9lbs. as cost of 15lbs. bears to cost of 9lbs.; thus,—

15lbs. : 9lbs. :: cost of 15lbs. : £1. 19s.

(4). 9lbs. will bear the same ratio to 15lbs. as cost of 9lbs. bears to cost of 15lbs.; thus,—

9lbs. : 15lbs. :: £1. 19s. : cost of 15lbs.

therefore, cost of 15lbs. $= \frac{39\text{s.} \times 15}{9} = 65\text{s.} = £3.\ 5\text{s.}$, *Ans.*

We should have found the same result (£3. 5s.) in each of the three previous forms, but as it is only to the last that special attention is

directed, the result has been given in this case only.* By comparing this fourth form with the example of which it is the solution, we obtain the following rule for the statement and solution of any Rule of Three question.

RULE. (1). Write the term which is like the required answer in the *third* place. (2). If the answer is to be *greater* than the third term, write the greater of the two remaining terms in the *second* place, and the lesser in the *first* place. (3). If the answer is to be *less* than the third term, write the lesser of the two similar terms in the *second* place, and the greater in the *first* place. (4). Reduce (if necessary) the first and second terms to the same denomination. (5). Multiply the second and third terms together, and divide the product by the first term; the result will be the answer required.

We may either leave the fourth place in the statement blank, or we may write the letter x, or we may write the word *answer*.

It should be noticed that in each of the four statements of Ex. 1, the two terms in any one ratio are of the *same* kind. The definition of ratio renders this essential. It should also be observed that 39s. is multiplied by the *abstract* fraction $\frac{1}{20}$, the value of any ratio being an abstract number.

Let us now consider the two following examples—

Ex. 2. If a certain number of men can build a wall 60yds. long in 5 days, what length of wall can they build in 9 days?

Ex. 3. If 8 men can build a wall in 6 days, how many men will be required to build it in 4 days?

In Ex. 2, it is proposed to increase the number of days from 5 to 9, and we are required to find what change this will make in the length of the wall. Now the number of workmen being the *same*, it is evident that they will build a *longer* wall in 9 days than in 5 days. Hence, an increase in the number of days gives a corresponding increase in the length of wall. When an *increase gives an increase*,

* The *first* form of statement deserves attention for the following reasons:—(1). It is the form used in all other branches of mathematics, and there is no reason beyond custom why it should differ in arithmetic. (2). It simplifies the rule for the statement of the terms by allowing each to be written down *in order*, the 1st term *first*, the 2nd, *second*, &c. According to this, the rule would take the following form.—*Write x in the 1st place, and the term like the answer in the 2nd place; then, if the answer is to be greater than the 2nd term, put the greater of the two similar terms third, and the lesser fourth; if the answer is to be less than the 2nd term, put the lesser of the two similar terms third, and the greater fourth, &c.* (3). The positions of x in the statement and in the resulting equation are symmetrical. (4). The answer *increases* as the ratio increases: the common form is the converse of this.

and a *decrease gives a decrease*, the question is said to be a *direct* proportion, or to belong to the *Rule of Three Direct*.

In Ex. 3, it is proposed to decrease the days from 6 to 4. Now the work being the *same*, it is evident that a *greater* number of men will be required to do it in 4 days than in 6 days. Hence, a decrease in the number of days gives an *increase* in the number of men. When a *decrease gives an increase*, and an *increase gives a decrease*, the question is then said to be an *inverse* proportion, or to belong to the *Rule of Three Inverse*.

When the terms of a question consist of tolerably simple numbers, the method of cancelling may generally be employed with advantage.

dys. dys. yds.
Ex. 2. 5 : 9 :: 60 : answer.

$$\text{therefore, answer} = \frac{9 \times \overset{12}{\cancel{60}}}{\underset{1}{\cancel{5}}} = 108\text{yds.}$$

dys. dys. men.
Ex. 3. 4 : 6 :: 8 : x.

$$\text{therefore, } x = \frac{6 \times 8}{4} = 12 \text{ men.}$$

When the numbers are complex it is better to multiply and divide in the usual way.

Ex. 4. If 4cwt. 3qrs. cost £5. 6s. 1d., what is the cost of 13cwt. 1qr.?

cwt. qrs. cwt. qr. £ s. d.
4 . 3 : 13 . 1 :: 5 . 6 . 1 : answer.
4 4 5
19 53 26 . 10 . 5
10
265 . 4 . 2
15 . 18 . 3
19)281 . 2 . 5(£14. 15s. 11d., *Ans.*
19
91
76
15
20
302(15s.
19
112
95
17
12
209(11d.
19
19
19

Or the reduction may be performed first, thus,—

4 cwt. 3 qrs. = 19 qrs.; 13 cwt. 1 qrs. = 53 qrs.; £5. 6s. 1d. = 1273d.

then. 19 qrs. : 53 qrs. :: 1273d. : answer

$$\begin{array}{r} 53 \\ \hline 3819 \\ 6365 \\ \hline 19\overline{)67469}\,(3551\text{d.} = £14.\ 15\text{s.}\ 11\text{d.}, \textit{Ans.} \\ 57 \\ \hline 104 \\ 95 \\ \hline 96 \\ 95 \\ \hline 19 \\ 19 \\ \hline \end{array}$$

Ex. 5. If $\frac{1}{3}$ of a ship be worth £2520, what is the value of $\frac{1}{4}$ of $\frac{4}{5}$ of the vessel?

$$\tfrac{1}{3} : \tfrac{1}{3} \text{ of } \tfrac{4}{5} :: £2520 : x$$

therefore, $x = \dfrac{2520 \times \frac{1}{3} \times \frac{4}{5}}{\frac{1}{3}} = \dfrac{2520 \,.\, 1 \,.\, 5 \,.\, 8}{7 \,.\, 3 \,.\, 6} = £800$, *Ans.*

The pupil should always be required to give the *reasons* for his statement of the question; thus, in Ex. 4,—

(1). £5. 6s. 1d. : answer, because the answer required is *money*.

(2). 4 cwt. 3 qrs. : 13 cwt. 1 qr., because 13 cwt. 1 qr. will cost more than 4 cwt. 3 qrs., and therefore the greater term is in the second place according to the rule.

Note.—Any Rule of Three question may be worked by the method given on page 53; the method there given is, however, more adapted to questions which do not contain complicated numbers. It will be useful practice to work all the simpler questions in the following exercises *both* ways.

Ex. 47.

1. If 8 lbs. of tea cost £1. 16s., what is the cost of 12 lbs.?

2. If 12 horses plough 66 acres in 7 days, how many acres will 16 horses plough in the same time?

3. How many yards of cloth may be bought for £5. 12s. 6d., at the rate of £9. 7s. 6d. for 15 yds?

4. In what time will 16 men complete a piece of work which requires the labour of 12 men for 8 days?

5. How many men will be required to finish a piece of work in 15 days which 18 men can finish in 10 days?

6. If the price of the 4lb. loaf be 5½d. when wheat is 49s. per *quarter*, what ought to be the price when wheat is 56s.?

7. If 4 st. of sugar be worth 5 lbs. of tea, how many pounds of tea are worth 10 st. of sugar?

8. When wheat is 50s. per quarter the sixpenny loaf weighs 4½ lbs.; what ought to be the weight when wheat is 45s.?

9. Find the cost of 1½ st. of sugar at £3. 0s. 8d. per cwt.

10. If the rent of a farm containing 308 acres is £847, how many acres ought to be rented for £1188?

11. What ought to be the railway fare for 108 miles at the rate of 11s. 6d. for 69 miles?

12. If 9 cwt. 3 qrs. of bar iron cost £10. 14s. 6d., how much ought to be bought for £62. 14s.?

13. A bankrupt pays 15s. 6d. in the pound, and his assets amount to £1736: find the amount of his debts.

14. A bankrupt pays 9s. 6d. in the pound, his debts being £665: find the amount of his assets.

15. A bankrupt's debts amount to £1232 and his assets to £1001; what dividend in the pound can he pay?

16. A bankrupt pays 14s. 6d. in the pound; what sum will be lost on a debt of £850?

17. The side of a square field measures 132 yds.; what ought to be the rent at £3. 15s. per acre?

18. A cubical block of marble whose edge measures 2 ft. weighs 9 cwt. 3 qrs.; find the weight of a cube of the same material, whose edge is 4 ft.

19. A gentleman's net income after paying an income tax of 8d. in the pound is £696: find his gross income.

20. If a hide of leather weighing 42 lbs. cost £1. 9s. 9d., what ought to be the value of 115 hides averaging 36 lbs. each?

21. The receipts of a railway company amount to £87,500 a-year, what dividend can they pay on a £100 share, the capital being £2,000,000?

22. Find the cost of 13 cwt. 2 qrs. 21 lbs. of sugar at £1. 19s. 8d. per cwt.

23. If ⅞ of a ship be worth £1840, what is the value of ¾ of it?

24. If a gentleman's income be reduced from £1000 to £956, 5s. by the payment of an income tax, what is the tax in the pound?

25. How many yards of cloth at 16s. 6d. per yard should be given in exchange for 22 yards at 19s. 6d. per yard?

26. If 13 men can do a piece of work in 18 days by working 10 hrs. a day, in what time will they finish it by working 12 hrs. a day?

27. Find the cost of 75 lbs. of coffee at 7c. 2m. per pound.

28. A Russian silver rouble is worth 3s. 1½d.; how many roubles should be given in exchange for £9. 4s. 9d.?

29. If ·625 cwt. of sugar cost £1. 15s., what is this per pound?

30. The tax on a property at 9½d. in the pound amounts to £611. 6s. 10½d.; what is the value of the property?

31. Find a fourth proportional to 2, 2½, and 3½.

32. The net rental of an estate, after deducting a property tax of 8d. in the pound, is £1933. 6s. 8d.; what is the gross rental?

33. If 3¼ yds. of silk cost £1. 19s., what is the value of 5⅝ yds.?

34. If a garrison of 750 soldiers have provisions to last 33 days, how long will the provisions last if they receive a reinforcement of 350 men?

35. If the carriage of 8 cwt. 3 qrs. for 113 miles be £3. 7s. 1d., what ought to be the carriage of 4 tons 2 cwt. 2 qrs. 21 lbs. for the same distance?

36. If 2 cwt. 3 qrs. 9 lbs. of sugar cost £4. 19s. 0¾d., what quantity may be bought for £10. 10s. 3¾d.?

37. If the rent of a farm containing 124 ac. 7 po. is £351. 9s. 1½d., what ought to be the rent of 281 ac. 3 ro.?

38. A merchant bought a cask of wine containing 95 gals. for £86. 13s. 9d., and after retaining 5 gals. for his own use he sold the remainder for £87. 15s.; what was his profit per gallon?

39. A coal merchant gives 10½ tons of coals and pays £12. 19s. in exchange for 6½ tons of hay at £3. 17s. per ton; what ought to be the price of a ton of coals?

40. A and B enter into partnership, their respective capitals being £645 and £755: their profits at the end of a year amount to £350; by how much will B's share exceed A's?

41. How much tea at 3s. 6d. per lb. ought to be given in exchange for 2 cwt. 1 qr. of sugar at 5½d. per lb.?

42. A workman's daily wages amount to £70. 5s. in a year of 52 weeks, but by working an extra quarter day for 11 weeks his wages amount to £73. 19s. 3d. at the end of the year: find the amount of his wages per day.

43. If ⅗ of a ship is sold for £387, what ought to be the value of ⅘ of the remaining share?

44. What number bears the same ratio to ·0̇3 as 4·641 bears to 2·73?

45. Find a first, second, and fourth proportional to 6, 8, and 12.

46. If eggs are sold at 6s. 8d. a hundred, how many ought to be bought for the same money at the rate of 3 more in a shilling's worth?

47. When wheat is 55s. per quarter how much ought to be given for 3 bus. 2 pks.?

48. If 9 men or 24 boys can earn £2. 8s. per day, what ought to be the united daily earnings of 12 men and 13 boys?

COMPOUND PROPORTION.

75. A ratio is said to be *compounded* of two or more other ratios when its value is equal to the values of the *other ratios* multiplied together.

An equality between the value of one ratio and the compound value of two or more others constitutes a *Compound Proportion.*

The ratio 6 : 8 = 5 : 4 multiplied by 9 : 15.

For, $\frac{6}{8}=\frac{3}{4}$; and, $\frac{5}{4}\times\frac{9}{15}=\frac{3}{4}$.

In arithmetic, this equality is usually expressed thus,—

$$\begin{matrix}5 : 4\\9 : 15\end{matrix} :: 6 : 8$$

In this statement all the terms are called *proportionals;* 4, 15, and 6, being distinguished as the *means,* and 5, 9, 8, as the *extremes.*

76 The continued product of the extremes = continued product of means

For, $\frac{5}{4}\times\frac{9}{15}=\frac{6}{8}$; therefore, multiplying by $4\times15\times8$ we have, $5\times9\times8=4\times15\times6$.

From this equality we can find any one term, if the remaining terms are known: for, any single *extreme* = the continued product of the means divided by the product of the remaining extremes; and any single *mean* = the continued product of the extremes divided by the product of the remaining means.

In arithmetical questions the term which it is required to find is generally made to occupy the place of the 8 in the above statement, that is, *the right hand extreme,* or fourth term. The required term might, however, be made to stand in any other place, and for the reasons given on page 110, it might advantageously stand as the *first* term by transposing the order of the statement.

If, in any one of the ratios on the left, the second term or *consequent* is greater than the first term or *antecedent,* this will tend to make the fourth term greater than the third; and conversely, if less, less. The fourth term, however, will only be really greater or less than the third, according as the continued product of *all* the terms standing in the second place is greater or less than the product of those in the first place.

In any question involving concrete numbers, it will be found that one of the given numbers is of the same kind as the required answer, and that the rest may be arranged in pairs, those in each pair being of the same kind. As has already been stated, the two terms of any ratio must be of the same kind.

From the above considerations we obtain the following:—

RULE. (1). Write the term which is like the answer as the *third* term of the statement. (2). Take each pair of the remaining terms in succession, reducing them

(if necessary) to one and the same denomination, and compare them with the third term and the required answer as in simple proportion, *on the supposition that all other things are equal:* write the greater term of any pair in the *second* place and the lesser in the *first* place, if the answer is to be *greater* than the third term; and the lesser term of any pair in the *second* place and the greater in the *first* place, if the answer is to be *less* than the third term. (3). Divide the continued product of the means by the product of the terms on the left: the quotient will be the required answer.

When the given numbers are comparatively simple the process of cancelling may generally be employed with advantage.

Ex. 1. If 7 men mow 10 acres in 8 days, how many acres will 12 men mow in 14 days?

(1). 10 ac. as the third term, because required answer is *acres*.

(2). 7 men : 12 men, because 12 men will mow more than 7 men.

(3). 8 days : 14 days, because more may be mown in 14 days than in 8 days.

Hence the statement will be,—

$$\begin{matrix} 7 : 12 \\ 8 : 14 \end{matrix} :: 10 : \text{answer.}$$

Therefore, answer $= \dfrac{\overset{3}{\cancel{12}} \times \overset{2}{\cancel{14}} \times 10}{\underset{1}{\cancel{7}} \times \underset{4}{\cancel{8}}} = 30$ *acres*.

This, or any other compound proportion question, might be worked as two or more simple proportion questions, thus,—

$$7 : 12 :: 10 : x$$

$$\text{therefore, } x = \frac{12 \times 10}{7} = \frac{120}{7} \text{ acres.}$$

$$\text{again, } 8 : 14 :: \frac{120}{7} : \text{answer.}$$

$$\text{therefore, answer} = \frac{14 \times 120}{8 \times 7} = 30 \text{ acres.}$$

Hence the application of the principles of compound proportion to *questions like* the above is often termed the *Double Rule of Three.*

Ex. 2. If the carriage of 1 ton 16 cwt. 2 qrs. 7 lbs. for 85 miles cost £2. 17s. 5½d., how far ought 3 tons 9 cwt. 7 lbs. to be carried for two-thirds of that sum ?

1 ton 16 cwt. 2 qrs. 7 lbs. = 4095 lbs.
3 tons 9 cwt. 7 lbs. = 7735 lbs. ; then,

(1). 85 miles is the third term because the answer is to be miles.
(2). 7735 lbs. : 4095 lbs. because the *greater* the weight to be carried the *less* should be the distance.
(3). The money being given in the ratio of 2 to 3 it is not necessary to take the sum in the question ; we therefore say 3 : 2 because the *less* the cost the *less* the distance. Hence,—

$$\begin{matrix} 7735 : 4095 \\ 3 : 2 \quad :: 85 : \text{answer} \end{matrix}$$

$$\text{therefore, answer} = \frac{4095 \times 2 \times 85}{7735 \times 3} = 30 \textit{ miles}.$$

In this, or in any other statement, the ratios on the left may be compounded as follows,—

$$7735 \times 3 : 4095 \times 2 :: 85 : \text{answer}.$$

which is equivalent to a simple proportion statement.

Ex. 3. If 9 men build a wall 130 yds. long, 6 ft. high, and 18 in. thick, in 7½ days of 12 hrs. each, in how many days will 12 men build a wall 100 yds. long, 13 ft. high, and 16 in. thick, working 10 hrs. a day ?

7½ is the third term because the answer is to be *days*.
12 men : 9 men because the *more* men the *fewer* days.
130 yds. : 100 yds. because the *shorter* the wall the *fewer* days.
6 ft. : 13 ft. because the *higher* the wall the *more* days.
18 in. : 16 in. because the *less* the thickness the *fewer* days.
10 hrs. : 12 hrs. because the *shorter* the day the *more* days required.

Hence the statement,

12 : 9
130 : 100
6 : 13 :: 7½ : answer
18 : 16
10 : 12

The required number of days is—

$$\frac{9 \cdot 100 \cdot 13 \cdot 16 \cdot 12 \cdot 7\frac{1}{2}}{12 \cdot 130 \cdot 6 \cdot 18 \cdot 10} = \frac{9 \cdot 100 \cdot 13 \cdot 16 \cdot 12 \cdot 15}{12 \cdot 130 \cdot 6 \cdot 18 \cdot 10 \cdot 2}$$

= 10 days, by cancelling.

It must be remembered that if any two terms forming a ratio are not in the same denomination, they must be reduced to the same denomination before working the question. The answer will be of the same kind and denomination as the third term.

It will form a useful exercise for the scholars to write down the reasons for the statement as given above.

Ex. 48.

1. If a man walk 102 miles in 4 days of 12 hrs. each, how far would he walk in 3 days of 10 hrs. each?

2. How many horses will be required to plough 12 acres in 9 days, if 6 horses plough 14 acres in 7 days?

3. If 7 cwt. be carried 54 miles for a guinea, how far ought 11 cwt. to be carried for a shilling more?

4. How many men will earn £15 in 5 days, if 7 men can earn £14 in 8 days?

5. If 18 men earn £22. 10s. in 9 days, what sum will 15 men earn in 4 days?

6. How far ought 91 second class railway passengers to travel for the same sum as is required for 117 third class passengers to travel 56 miles, the second class fare being 1½d. per mile, and the third class fare 1d.?

7. The diameters of the hind and fore wheels of a coach are as 3 : 2; if the fore wheel makes 78 revolutions in going a certain distance, how many revolutions will the hind wheel make in twice the distance?

8. What weight ought to be carried 59½ miles for £1. 16s. 0½d., the carriage of 1 ton 15 cwt. 7 lbs. for 29 miles being £6. 19s. 6¾d.?

9. If the 4½ lb. loaf costs 9d. when wheat is 56s per quarter, what should be the price of wheat when the 4 lb. loaf costs 6d.?

10. If a capital of £3200 gain £192 in 8 months, in what time will £2500 gain £75?

11. If £35 keep a family of 7 persons for 15 weeks, how long should 40 guineas keep 9 persons at the same rate?

12. The revenue derived from the duty on an article is £720,000, but by taking off one-third the duty the consumption is doubled: find the increase in the revenue.

13. If 12 horses plough 32 acres in 16 days, in how many days will 21 horses plough 35 acres?

14. If 16 men build a wall 128 yds. long in 2⅔ days of 10½ hrs. each, how many men will be required to build a similar wall 90 yds. long in 7 days of 9 hrs. each?

15. If 7 lbs. of tea are worth 48 lbs. of sugar, and 16 lbs. of sugar are worth 3½ lbs. of coffee, how many pounds of coffee should be given in exchange for 44 lbs. of tea?

16. The expenses of a family of 7 persons for 11 months amount to £289. 14s. 3d.; what sum will keep 9 persons for 5 months at the same rate?

17. If by reducing the duty on an article from 1s. 3d. to 1s. the revenue be increased from £220,000 to £264,000, in what proportion is the consumption increased?

18. If it cost £6. 16s. 1½d. to cover a room with carpet ⅞ of a yard wide at 2s. 9d. per yd., how much may be saved by using carpet 1¼ yds. wide at 3s. 4d. per yd.?

19. If 3 tons 5 cwt. 1 qr. 21 lbs. be carried 173 miles for a certain sum, how far ought 13 cwt. 3 qrs. 17 lbs. to be carried for the same sum, when the rate of carriage is one-sixth more?

20. If 20 men dig a trench 53 yds. long, 11 ft. wide, 7 ft. deep, in 3 days of 12 hours each, how many hours per day must 36 men work to dig a trench twice as long, one-fourth as wide, and half as deep again, in 5 days?

21. The revenue derived from the duty on an article is increased one-third by doubling the duty: in what proportion is the consumption increased or diminished?

22. If 3 men mow a field of 6 ac. 2 ro. 33 po. in 10 days of 9 hrs. each, how many hours a day must 5 men work to mow a field 407 yds. long and 319 yds. broad in 24 days?

23. If when 2 cwt. of wheat is sold for £2. 16s. four and a half pounds of bread can be bought for 6d., what ought to be the weight of the fourpenny loaf when wheat is selling at 12s. 6d. per cental?

24. If 4 tons of hay serve 7 cows for 50 days, how long will 6 tons of hay serve 9 horses, supposing 5 cows to eat as much as 3 horses?

25. A lends B £875 for 9 months; for how long should B lend A 500 guineas to return the obligation, when the rate of interest is half as much again?

26. If 16 horses eat 24½ bushels of oats in 7 days when the price is 33s. per quarter, what sum will be required to keep 22 horses for 8 days, when the price is 35s. per quarter?

27. A can mow 1½ acres of grass in 15 hrs. and B can mow 3½ acres in 28 hrs.; how many acres will B mow while A is mowing 9⅝ acres?

28. If 6 horses cost as much as 13 oxen, and 4 oxen cost £51, how much should be given for 16 horses?

29. A walks 3 miles while B walks 4, and B walks 5 miles while C walks 6; how many miles will A walk while C walks 44?

30. The cost of painting the walls of a room 46 ft. long, 20 ft. broad, and 16 ft. high, is £11; find the cost of painting the walls of another room 22 ft. long, 17 ft. broad, and 14 ft. high, at the same rate.

31. A besieged garrison consisting of 663 soldiers has provisions to last 33 days, allowing each man 20 oz. a day; what reinforcement can they receive so that by allowing each man 12 oz. a day the provisions may last 39 days?

32. Twenty dozens of port wine cost as much as 22½ doz. of sherry, and 36 doz. of sherry cost as much as 16 doz. of champagne: if port be 54s. per dozen, what is the price of champagne?

33. There are two families, consisting of 5 and 8 persons respectively, whose consumption of tea, bread, and meat, is proportional to their numbers; when tea is 4s. 6d. per lb., meat 6d., and the sixpenny loaf weighs 3½ lbs., the expenses of the larger family for these three articles amount to £2. 5s. per week. Find the expenses of the other family for 10 days, when tea is 4s. 4d. per lb., meat 7½d., and the eightpenny loaf weighs 6½ lbs.

INTEREST.

77. We shall now show the application of the principles of proportion to various rules relating to monetary transactions.

Interest is money paid for the use of money.

The interest is paid by the *borrower* to the *lender*.

The sum lent is called the *Principal*.

The principal is lent for a given *Time*.

The *Rate per cent* is the interest paid on £100 for 1 year.

The *Amount* for a given time is the principal *together with* the interest for that time.

When the interest is paid on the original principal from year to year (or any other fixed period), it is called *Simple Interest*.

When the interest, instead of being paid from year to year (or any other fixed period), is added to the principal, to form a new principal for the next period, it is called *Compound Interest*.

Simple interest must always be understood, except the contrary be stated.

Example. Find the interest on £435 for 5 yrs. at 4 per cent.

This question is equivalent to the following question in compound proportion,—

If the interest on £100 for 1 year be £4, what is the interest on £435 for 5 years?

£4 is the third term because answer required is *interest*.

£100 : £435, because the *greater* the principal the *greater* the interest.

1 yr. : 5 yrs., because the *longer* the time the *greater* the interest.

Hence the following statement and solution,—

$$\begin{matrix}£100 : £435 \\ 1\text{ yr.} : 5\text{ yrs.}\end{matrix} :: £4 : \text{interest.}$$

$$\text{therefore, interest} = \frac{435 \times 5 \times 4}{100 \times 1} = £87.$$

If we take I=interest, P=principal, T=time, R=rate per cent., and A=amount, the statement becomes—

$$\begin{matrix} 100 : P \\ 1 : T \end{matrix} :: R : I$$

From this statement we may deduce the following formulæ,—

1. $I=\frac{P\times T\times R}{100}$ 2. $A=P+I$ 3. $P=\frac{I\times 100}{T\times R}$

4. $T=\frac{I\times 100}{P\times R}$ 5. $R=\frac{I\times 100}{P\times T}$ 6. $P=\frac{A\times 100}{100+T\times R}$

7. $T=\frac{(A-P)\times 100}{P\times R}$ 8. $R=\frac{(A-P)\times 100}{P\times T}$

These forms may respectively be translated into the following,—

RULES. 1. *To find the Interest.*—Multiply the principal, time, and rate together, and divide the product by 100.

2. *To find the Amount.*—Add the principal and interest together.

3. *To find the Principal when interest, time, and rate are given.*—Multiply the interest by 100, and divide the product by the time multiplied by the rate.

4. *To find the Time when the principal, interest, and rate are given.*—Multiply the interest by 100, and divide the product by the principal multiplied by the rate.

5. *To find the Rate when the principal, interest, and time are given.*—Multiply the interest by 100, and divide the product by the principal multiplied by the time.

6. *To find the Principal when the amount, time, and rate are given.*—Multiply the amount by 100, and divide the product by 100 added to the product of the time and rate.

7. *To find the Time when the principal, amount, and rate are given.*—Subtract the principal from the amount, multiply the remainder by 100, and divide by the product of the principal and rate.

8. *To find the Rate when the principal, amount, and time are given.*—Subtract the principal from the amount, multiply the remainder by 100, and divide by the product of the principal and time.

Since $A-P$=interest, Rules 7 and 8 are equivalent to 4 and 5.

The following eight questions are respectively examples of the foregoing eight Rules. When the given numbers are comparatively simple the method of cancelling may be employed with advantage.

Ex. 1. Find the interest on £737. 10s. for 3 yrs. at 4 per cent.

£737 . 10s.
3
2212 . 10
4
100)88 . 50 . 0
20
10 . 00 *Ans.*, £88. 10s.

Or thus, $I=\frac{737\frac{1}{2}\times 3\times 4}{100}$

$=\frac{1475\times 3\times 4}{2\times 100}=$£88. 10s.

Ex. 2. Find the amount of £106. 9s. 2d. for 225 days at 4½ per cent.

£ s. d.
106 . 9 . 2
4½
425 . 16 . 8
53 . 4 . 7
£4 . 79 . 1 . 3
20
15 . 81s.
12
9 . 75d.
4
3 . 00f.

Here we first find the interest for 1 year which=£4. 15s. 9¾d. Then to find the interest for 225 days we have—

dys. dys. £ s. d.
365 : 225 :: 4 15 9¾ : interest required.

whence interest=£2. 19s. 0¾d. which added to £106. 9s. 2d. gives £109. 8s. 2¾d. as the amount required.

Ex. 3. What sum will produce £91 interest in 7 yrs. at 3½ per cent.?

Here we have to find the principal; and by Rule 8,

$$P=\frac{£91\times 100}{7\times 3\frac{1}{2}}=\frac{£91\times 100\times 4}{7\times 13}=£400\ \textit{Ans.}$$

Ex. 4. In what time will £314 produce £39. 5s. interest at 3½ per cent.?

By Rule 4, $T=\frac{39\frac{1}{4}\times 100}{314\times 3\frac{1}{2}}=\frac{157\times 100\times 8}{4\times 314\times 25}=4$ yrs. *Ans.*

Ex. 5. At what rate per cent. will £685. 16s. produce £57. 3s. interest in 3 yrs. 4 months?

£685. 16s. = 13716s.; £57. 3s. = 1143s.; 3 yrs. 4 mo. = $\frac{10}{3}$ yrs.

By Rule 5, $R = \frac{1143 \times 100 \times 3}{13716 \times 10} = 2\frac{1}{2}$ per cent. *Ans.*

Ex. 6. What sum will amount to £98. 6s. 3d. in 7 yrs. at 3 per cent.?

$7 \times 3 = 21$, 100, 121

£ s. d.
98 . 6 . 3
$10 \times 10 = 100$
983 . 2 . 6
10
121)9831 . 5 . 0(£81. 5s.
968
151
121
30
20
605
605(5s.

Ans. £81. 5s.

Ex. 7. In what time will £120 amount to £150 at 7½ per cent.?

By Rule 7, $T = \frac{(150-120) \times 100}{120 \times 7\frac{1}{2}} = \frac{30 \times 100 \times 2}{120 \times 15} = 3\frac{1}{3}$ yrs. *Ans.*

Ex. 8. At what rate per cent. will £80 amount to £90 in 2½ yrs.?

By Rule 8, $R = \frac{(90-80) \times 100}{80 \times 2\frac{1}{2}} = \frac{10 \times 100 \times 2}{80 \times 5} = 5$ per cent. *Ans.*

Ex. 49.

Find the interest on—

1. £335 for 4 years at 5 per cent.
2. £436 for 5 years at 8 per cent.
3. £337. 10s. for 6 years at 4 per cent.
4. £862. 10s. for 2 years 8 months at 4½ per cent.
5. £593. 15s. for 3 yrs. 4 mo. at 2½ per cent.
6. £138. 7s. 11d. for 288 days at 3¾ per cent.

Find the amount of—

7. £845 for 3 years at 4 per cent.
8. £1716 for 9 yrs. 6 mo. at 3½ per cent.
9. £456. 5s. from April 14, to September 19, at 3 per cent.

What sum will produce—

10. £77 interest in 8 yrs. at 2½ per cent.?
11. £17. 10s. interest in 7 yrs. at 3¾ per cent.?
12. £4. 0s. 8d. interest in 5½ yrs. at 2¾ per cent.?

In what time will—

13. £145 produce £87 interest at 3 per cent ?

14. £327. 10s. produce £27. 5s. 10d. interest at 2½ per cent. ?

15. £1124. 3s. 9d. produce £93. 13s. 7½d. interest at 3⅓ per cent. ?

At what rate per cent. will—

16. £292 produce £73 interest in 8 years ?

17. £576. 10s. produce £57. 13s. interest in 2½ yrs. ?

18. £161. 11s. 9½d. produce £2. 1s. 5¼d. interest in 117 days ?

What sum will amount to—

19. £140 in 3 yrs. at 4 per cent. ?

20. £373. 10s. in 7 yrs. at 3½ per cent. ?

21. £431. 12s. 9¾d. in 5½ yrs. at 3⅓ per cent. ?

In what time will—

22. £60 amount to £70 at 3½ per cent. ?

23. £180 amount to £184. 12s. 9¾d. at 2½ per cent. ?

24. £640 amount to £673 at 4⅕ per cent. ?

At what rate per cent. will—

25. £750 amount to £845 in 4¾ years ?

26. £178. 15s. amount to £185. 18s. in 146 days ?

27. £235. 10s. 5d. amount to £242. 11s. 8¾d. in half a year ?

28. In what time will £97 double itself at 5 per cent. ? In what time will *any* sum double itself at the same rate ?

29. In what time will any sum double itself at 4 per cent. ?

30. At what rate per cent. will any sum double itself in 50 years ? At what rate in 16 years ?

31. What sum will amount to as much in 9 years at 5½ per cent. as £550 does in 7 years at 6 per cent. ?

32. At what rate per cent. will £65 produce the same amount of interest in 7½ years as £75 does in 5¼ years at 3½ per cent. ?

78. *Compound Interest.*—RULE. Find the interest for the first year by the preceding rule, and add it to the principal to form the principal for the second year: add the interest of the second year to the principal for that year to form the principal for the third year: proceed in the same way for each successive year. Add together the interests for the several years, and the result will be *the compound* interest required.

Ex. 1. Find the compound interest on £240 for 2 years at 5 per cent.

$$\text{interest for 1st year} = \frac{240 \times 5}{100} = £12.$$

$$\text{interest for 2nd year} = \frac{252 \times 5}{100} = £12.\ 12s.$$

therefore, compound interest = £12 + £12. 12s. = £24. 12s.

Commission is money charged by an agent for buying or selling goods for another.

Brokerage is similar to commission but relates chiefly to money transactions, such as the buying and selling of shares, &c.

Insurance is a contract for securing the value of property which is subject to risk by the payment of a certain per centage or *Premium* on its value.

Life Assurance is a contract by which one party A, on receiving a certain sum or *premium* annually from B, undertakes in the event of B's death to pay a specified sum to his heirs. The document entitling B to this claim upon A is called a *Policy of Assurance.*

Ex. 2. Find the brokerage at $\frac{1}{8}$ per cent. for buying 20 railway shares at £80 each.

$$\frac{80 \times 20 \times \frac{1}{8}}{100} = \frac{80 \times 20 \times 1}{100 \times 8} = £2\ \textit{Ans.}$$

Ex. 3. What sum must be insured at 6 per cent. on goods worth £1880, so that in case of loss both the value of the goods and the premium may be recovered?

By insuring the goods at their actual value the premium will be lost since the owner will receive £1880 only. If, however, every (£100 − £6) be insured for £100, the value of the goods and premium will both be recovered.

Hence, £94 : £100 :: £1880 : required sum

$$\text{therefore, required sum} = \frac{100 \times 1880}{94} = £2000.$$

Ex. 50.

1. Find the compound interest on £300 for 2 yrs. at 5 p.c.
2. Find the compound interest on £1600 for 3 yrs. at $2\frac{1}{2}$ p.c.

3. Find the amount of £800 for 3 years at 7½ per cent., compound interest.

4. Find the compound interest on £2000 for 4 years at 5 per cent.

5. What is the commission at 2½ per cent. for buying goods worth £3300?

6. Find the brokerage at ¼ per cent. for buying 16 railway shares at £90 each.

7. What is the premium at 7½ per cent. for insuring a cargo of goods worth £1950?

8. What sum must be insured at 4 per cent. on goods worth £1680, so that in case of loss both the value of the goods and the premium may be recovered?

DISCOUNT.

79. *Discount* is an allowance made for the payment of money before it is due.

The following is one of the most frequent cases in which discount occurs:—A buys £100 worth of goods from B, but instead of paying ready money he gives B a written form, called a *Bill* or *Note of Hand*, in which he promises to pay £100 at the end of a given time (say 3 months), either to B or to any one else who may hold (that is, be in possession of) the Bill at that time. As it may be inconvenient for B to wait 3 months for the money he takes the Bill to a banker, or bill-broker, who, if A's credit is known to be good, pays B the amount of the Bill, deducting a certain per centage or discount: this is called *discounting the Bill*. B has no further claim upon A; the banker having in fact *bought* the claim, it is to him that A is now indebted. The banker or bill-broker may in his turn part with the Bill to some one else, and it may, and probably would, pass through several hands before it returns to A. But at any intermediate time A is indebted to the amount of £100 to whoever may hold the Bill. It will be seen, then, that a Bill is a *negotiable* article.

Suppose the banker charges discount at 8 per cent. per annum, he would reckon thus,—"the interest of £100 for 3 months at 8 per cent. is £2," and would deduct £2 from the amount of the Bill, paying B £98 only. *Banker's* or *Mercantile* discount is therefore equivalent to *interest* on the amount discounted. The true statement of the question, however, is not, "what is the interest on £100 for 3 months at 8 per cent?" but, "what sum will amount to £100 in 3 months at 8 per cent.?" This would be found to be £98. 0s. $9\frac{1}{17}$d., and it is the difference between this sum and £100, namely £1. 19s. $2\frac{16}{17}$d. that constitutes the *true discount*. The first result (£2) is in reality the discount on £102 for 3 months at 8 per cent. In practice, mercantile discount is always reckoned; and the £98 which B receives is called the *present worth* of the Bill.

Three days, named *days of grace*, are allowed from the time when a *Bill becomes nominally* due before it is *legally* due. A Bill due on

July 1st is legally due on July 4. The day on which a Bill is drawn is not reckoned in discounting it.

Mercantile discount being equivalent to interest is found by the rule for interest; and the present worth is found by deducting the interest from the amount of the Bill.

Ex. 1. Find the discount on and present worth of a Bill for £350, due 7 months hence, at 6 per cent. per annum.

$$\text{discount} = \frac{£350 \times 7 \times 6}{100 \times 12} = £\frac{49}{4} = £12.\ 5s.$$

present worth = £350 − £12. 5s. = £337. 15s.

To find the true discount on any sum for a given time at a given rate.—RULE. Find the interest on £100 for the given time at the given rate, and state thus,—

£100 + its interest : given sum : : interest on £100 : discount required.

Ex. 2. Find the true discount on a bill for £160, due 3 months hence, at 6 per cent. per annum.

interest on £100 for 3 mo. at 6 p.c. = £1. 10s.

therefore, £101½ : £160 : : £1½ : discount required.

$$\text{whence, discount} = \frac{160 \times 3 \times 2}{203 \times 2} = £2.\ 7s.\ 3\tfrac{99}{203}d.$$

To find the Present Worth of any sum due in a given time at a given rate.—RULE. Find the interest on £100 for the given time at the given rate, and state thus,—

£100 + its interest : given sum : : £100 : present worth required.

Ex. 3. Find the present worth of a bill for £570, due 4 months hence, at 7½ per cent. per annum, true discount.

interest on £100 for 4 mo. at 7½ p.c. = £2. 10s.

therefore, £102½ : £570 : : £100 : present worth required.

$$\text{whence, present worth} = \frac{570 \times 100 \times 2}{205} = £556.\ 1s.\ 11\tfrac{1}{41}d.$$

Ex. 51.

Find the Mercantile Discount on—

1. £675 due 1 year hence at 3 per cent. per annum.
2. £960 6 months 5
3. £57. 10s. 3 months 6
4. £127. 15s. 90 days 3½
5. £31. 18s. 9d... 50 days 8½
6. £720. 4s. 5 months 7½

Find the Present Worth of the following bills, mercantile discount being allowed—

7. £220 due 4 months hence at 6 per cent. per annum.
8. £740 9 months 4
9. £436. 10s. 8 months 6¼
10. £262. 10s. 7 months 5½
11. £237. 5s. 256 days 8¾
12. £150. 11s. 3d. . 320 days 7⅛

Find the true Discount on—

13. £177. 13s. 4d. due 6 months hence at 5 per cent. per annum.
14. £169. 8s. 4 months 8
15. £76. 1s. 9 months 7½
16. £612. 15s. 8 months 4½
17. £168. 7s. 6d. 73 days 5⅛
18. £1340 29 days 7

Find the Present Worth of the following bills, true discount being allowed,—

19. £656 due 6 months hence at 5 per cent. per annum.
20. £690 1 year 3½
21. £121. 5s. 6d. 3 months 4½
22. £31. 15s. 0½d. 11 months 6¾
23. £1500 60 days 7½
24. £209. 18s. 1½d. .. 98 days 8½

25. A bill for £250 drawn on May 15th at 107 days, is discounted on June 18th at 7½ per cent.: find the discount.

26. A bill for £127. 15s. drawn April 26th at 178 days, is discounted on July 23rd at 3½ per cent.: what sum will the holder of the bill receive?

27. A tradesman marks his goods at 5 per cent. above the ready money price in order to allow 6 months credit: if the credit price of an article be £3. 15s., what sum will buy it in ready money?

28. B draws on A for £547. 10s. on July 3rd at 66 days, and gets the bill discounted by a banker on July 14th at 4 per cent. Before the bill is due A becomes bankrupt and pays a dividend of 12s. 6d. in the pound: how much does the banker lose on the amount paid to B?

29. What sum in ready money will suffice for the payment of a debt of £385 due 8 months hence at 5½ per cent., true discount being reckoned?

30. A bill for £263. 10s. at 60 days is paid 23 days before it is due: what is the mercantile discount at 6½ per cent.?

31. A bill for £705. 15s. 7d. at 75 days is paid 46 days after date: what is the true discount at 4⅛ per cent.?

32. A piece of French silk, including an import duty of 7 per cent., costs £139. 2s. If the French manufacturer receives £123. 10s., what discount does he allow on the original price exclusive of the *duty?*

STOCKS.

80. *Stock* is the name given to money lent to the Government of a country. It is also used to designate the capital of large trading companies, such as Railway and Mining Companies.

The price of stock is always quoted at so much per £100 worth. From political and other causes the price is continually changing. When the price of £100 stock is £100, it is said to be at *Par*; when the price is more than £100, it is said to be at a *Premium*; and when the price is below £100, it is said to be at a *Discount*.

Stock is bought and sold by a broker, who charges $\frac{1}{8}$th per cent.; that is, he charges 2s. 6d. for every £100 stock he buys or sells, whatever the price may be.

Government stock is generally named according to the rate of interest; thus,—3 *per cents.*; $3\frac{1}{4}$ *per cents.*; 3 *per cents. reduced*; *New* 3 *per cents.*; &c. *Consols* is a contraction for Consolidated Fund.

The money article in the *Times* for June 27, 1860, contains the following statement:—"Consols were $93\frac{1}{2}$, $\frac{3}{8}$, and finally, $93\frac{3}{8}$, $\frac{1}{2}$." The meaning of this is that the price of £100 worth of the stock known as consols was from $93\frac{1}{2}$ to $93\frac{3}{8}$; that is, from £93. 5s. to £93. 7s. 6d. when business began, and from £93. 7s. 6d. to £93. 10s. at the close of the market.

If we include the brokerage, a person buying stock at $93\frac{1}{2}$ will have to pay $(93\frac{1}{2}+\frac{1}{8})$, or $93\frac{5}{8}$; and a person selling stock at $93\frac{1}{2}$ will only receive $(93\frac{1}{2}-\frac{1}{8})$, or 93. This is evident, since he has to pay the broker $\frac{1}{8}$ in each case. The brokerage need not be noticed in working the questions unless mentioned.

Examples.

1. How much 3 per cent. stock at $94\frac{1}{2}$ may be bought for £3780?

$94\frac{1}{2} : 100 :: £3780 :$ stock required.

whence, stock required $= £4000$ *Ans.*

2. What is the price of stock when £600 worth can be bought for £582?

$600 : 582 :: 100 :$ price required.

whence, price required $= 97$.

3. What money would a person receive by selling £4000 stock at $95\frac{7}{8}$, reckoning the usual brokerage.

deducting brokerage, the price is $95\frac{7}{8}-\frac{1}{8}=95\frac{3}{4}$.

then, $100 : 95\frac{3}{4} :: £4000 :$ sum required.

whence, sum required $= £3830$ *Ans.*

4. At what rate does a person receive interest who invests his money in the 3½ per cents. at 81¼ ?

81¼ : 100 :: 3½ : rate required.

whence, rate required = 4 per cent.

5. A person transfers £3745 stock from the 3 per cents. at 89¼ to the 3½ per cents. at 93⅝ ; find the alteration in his income.

Here we first find the quantity of 3½ per cent. stock, which he will purchase by the transfer. This will be less than £3745 in proportion as the price is more than 89¼ ; that is, the quantity of stock will be in the inverse ratio of the prices.

Hence, 93⅝ : 89¼ : : £3745 : stock required.

whence, stock required = £3570.

income in first case = £112. 7s. 0d.

income in second case = £116. 0s. 6d.

increase in income = £3. 13s. 6d., *Ans.*

The following rules for the statement of questions relating to stocks will be found useful.

1. *To find the quantity of stock purchased for a given sum.*

Price of stock : 100 : : sum invested : stock required.

2. *To find the sum realized by selling any quantity of stock.*

100 : price of stock : : stock sold : sum required.

3. *To find the price of stock when a certain quantity can be bought for a given sum.*

Stock bought : given sum : : 100 : price required.

4. *To find the income arising from the investment of a given sum in stock at a certain price, which pays a given rate of interest.*

Price of stock : sum invested : : given rate : : income required.

Note.—The income derived from a given quantity of stock is a question of simple interest.

5. *To find what sum must be invested in stock at a given price to produce a given income.*

Interest on £100 stock : given income : : price of stock : sum required.

6. *To find the rate at which interest will be received by buying stock at a given price.*

Price of stock : £100 : : interest on £100 stock : rate required.

7. *The transference of stock.*

Price of stock bought : price of stock transferred : : amount of stock transferred : amount of stock bought.

Note.—The *inverse* form of this last statement should be noticed.

Ex. 52.

How much stock can be purchased by investing—

1. £2730 in the 3 per cents. at 91?
2. £1295 in the 3½ per cents. at 92½?
3. £1131 in the New Threes at 94¼?
4. £1120. 10s. in the Old 3 per cents. at 93⅜?
5. £2872. 10s. in consols at 95⅝, brokerage ⅛ per cent.?
6. £585. 7fl. 5c. in Rupee 5½ per cents. at 97⅝, brokerage ⅛?

What is the price of stock when—

7. £1000 worth is bought for £920?
8. £2000 worth is bought for £1790?
9. £480 worth is bought for £449. 4fl.?

What sum will be realized by selling—

10. £550 stock at 91¼?
11. £3445 New Threes at 93⅞?
12. £4735 Brazilian 5 per cents. at 2⅛ premium?
13. £960 Turkish 6 per cents. at 33½ discount, brokerage ⅛?
14. £1790 Russian 4½ per cent. stock at ⅛ below par, reckoning brokerage ⅛?

Find the annual income derived from the investment of—

15. £527 in the 3 per cents at 93.
16. £2850 in the 3½ per cents. at 97½.
17. £2671 in Russian 5½ per cents. at 102¾.
18. 4403 guineas in the New Threes at 97½.
19. £1753. 5fl. in Turkish 6 per cents. at 62½, brokerage ½.

What sum must be invested in—

20. 3 per cents. at 93 to produce an annual income of £100?
21. 3½ per cents. at 88½ 819 guineas?
22. Spanish 5½ per cents. at 60½ 500 guineas?
23. 3¾ per cents. at 89⅜ 1000 guineas?
24. Brazilian 5 per cents. at 2¾ premium, brokerage ½, to produce an annual income of £1550?

At what rate per cent. will interest be received on money invested—

25. In the 3 per cents. at 90?
26. In the 3½ per cents. at 91?
27. In Railway shares paying 5½ per cent. at 87⅞, brokerage ⅛?
28. If a person transfers £2835 from the 3 per cents. at 89¼ to the 3½ per cents. at 94½, how much of the latter stock will he purchase by the transfer?
29. What amount of Brazilian 5 per cent. stock at 3½ premium will a person purchase by the transfer of £3712. 10s. Russian 4½ per cent. stock at 2⅜ discount?

30. Railway shares paying $6\frac{1}{4}$ per cent. are at 20 per cent. premium: what annual income will a person derive from the investment of £6126. 7s. 6d. in such shares, reckoning brokerage $\frac{1}{8}$?

31. A person transfers £8762. 10s. from the $3\frac{1}{4}$ per cents. at $97\frac{1}{8}$, to the 3 per cents. at $87\frac{1}{8}$; find the alteration in his income.

32. A person invests £1761. 15s. in the 3 per cents. at $97\frac{3}{4}$, and sells out when they have risen to *par*: what sum does he clear, reckoning brokerage $\frac{1}{8}$ both in buying and selling?

PROFIT AND LOSS.

81. Questions relating to the profits and losses attending trading transactions are solved by the application of the principles of proportion.

Ex. 1. Tea is bought at 2s. 11d. per lb.; at what price must it be sold so as to gain 20 per cent.?

2s. 11d. = 35d.; 100d. worth is to be sold for 120d.

hence, 100 : 35 :: 120 : price required.

whence, price required = 42d. = 3s. 6d.

Ex. 2. Sugar which costs 35s. per cwt. is sold at $4\frac{1}{2}$d. per lb.; what is the profit per cent.?

$4\frac{1}{2}$d. per lb. = 42s. per cwt. therefore, the gain on 35s. is 7s.;

hence, 35 : 100 :: 7 : profit required.

whence, gain required is 20 per cent.

Ex. 3. An article is sold at 5s. 6d. at a profit of 32 per cent.: find the prime cost.

5s. 6d. = 66d. The reasoning is—If the prime cost of an article which is sold at 132d. be 100d., what is the prime cost of an article which is sold at 66d.?

Hence, 132 : 66 :: 100 : prime cost required.

whence, prime cost = 4s. 2d.

Ex. 4. A grocer mixes tea at 4s. 6d. per lb. with tea at 3s. 4d., and sells the mixture at a profit of 25 per cent.: what is the selling price per lb.?

cost price per lb. = (4s. 6d. + 3s. 4d.) ÷ 2 = 3s. 11d. = 47d.

hence, 100 : 47 :: 125 : selling price.

whence, selling price = 4s. $10\frac{1}{4}$d.

Ex. 53.

1. A grocer buys a cask of currants weighing 11½ cwt. at £2. 11s. 4d. per cwt. and retails them at 7d. per lb.: what is his total profit?

2. How much per cent. is gained by selling sugar at 6d. per lb. which cost 5d.?

3. A horse is sold for 60 guineas at a loss of 15 per cent.: find the prime cost.

4. A merchant buys 60 gallons of brandy at 31s. per gallon, and sells the whole for 100 guineas: what is his profit per cent.?

5. Coals are sold at a guinea a ton at a loss of 16 per cent.; at what price must they be sold so as to gain 10 per cent.

6. A cwt. and a half of sugar is sold at 4½d. per lb. at a profit of 12½ per cent.: what is the total profit?

7. If 10 per cent. is lost by selling eggs at 18 for a shilling, how much per cent. is gained by selling them at 15 a shilling?

8. A chest of tea weighing 56 lbs. is sold for 3s. 6d. a pound at a loss of 8 per cent.; what is the loss on the whole?

9. A sells a horse to B at a profit of 10 per cent., and B sells the horse to C for £72. 12s. at a profit of 10 per cent.: what did A pay for the horse?

10. A French silk manufacturer supplies a wholesale firm in London with silk at 28 per cent. profit; the wholesale firm sell it for 3s. 2d. per yd. at a loss of 5 per cent.: find the cost per yd. to the manufacturer.

11. A person buys 25 railway shares at 83⅓, and sells them when they have advanced 12 per cent.: find the selling price per share and the total profit.

12. If 5 per cent. be lost by selling cloth at 9s. 6d. per yd., at what price must it be sold to gain 7½ per cent.?

13. Oranges are bought at 4s. 6d. a hundred, and sold at 9d. a dozen: what is the gain per cent.?

14. How much per cent. is cleared in buying stock at 3⅛ discount and selling it at 2⅛ premium?

15. A person buys £50 worth of goods, and sells half of them at a loss of 6¼ per cent.; for what sum must he sell the remainder so as to gain 25 per cent. on the whole?

16. A merchant buys a thousand quarters of wheat at 45s. per qr., and sells 300 quarters at a loss of 15 per cent. If he sells the remainder for £2081. 5s. how much per cent. will he gain on the whole?

17. A merchant sells cheese at 36s. per cwt. and thereby clears ⅛ of the cost price; if the price advances 2s. 6d. per cwt., how much will he clear per cent.?

18. If eggs are bought at 10 for 9d. and sold at the rate of 15 for 1s. 2d., how much is gained on 9s. worth?

19. A person buys 27 railway shares at £90 each, and afterwards 15 more at £92 each; if he sells the whole at a profit of 7½ per cent., how much will he clear altogether?

20. A person invests £1490 in stock at 93, and when the price has risen to $2\frac{5}{16}$ above *par* he sells out; what is his profit per cent., and how much does he clear altogether, reckoning ⅛ for brokerage both in buying and selling.

21. A sells goods to B at a profit of 7½ per cent.; B sells the goods to C at a loss of 10 per cent.; and C sells the same goods to D for £24. 3s. 9d., thereby clearing a profit of 25 per cent.: what did the goods cost A?

22. A wine merchant buys a pipe of wine at 10s. a gallon, and after losing one-seventh by leakage he sells the remainder at 12s. per gallon: how much does he gain on the whole, and how much per cent.?

23. A farmer buys sheep at the rate of £15 per dozen, and sells them at the rate of £35 per score; what is his total profit on 1150 sheep, and what per cent.?

24. A coal merchant buys 600 tons of coals at 18s. 6d. per ton; he sells one-third at a profit of 10 per cent., three-tenths of the remainder at a loss of 5 per cent., and the rest at a profit of 5 per cent. How much does he clear on the whole, and how much per cent.?

EXCHANGE AND FOREIGN TRADING.

82. *Foreign Exchange* is the transference of money accounts between different countries through the medium of Bills of Exchange.

A *Bill of Exchange* is similar in purport to an Inland Bill (Art. 79.), excepting that the transaction to which it refers is between different countries.

The *Course of Exchange* is the value of a fixed sum of money of one country as estimated in the money of another country. In London, £1 sterling is usually taken as the fixed sum, and in exchange for this from 25 to 25½ francs in French money may be obtained. The Course of Exchange is consequently *variable*.

When the Exchange is between our own country and another it is called *Direct*. When it is between two

foreign countries it is called *Cross Exchange.* When it is between our own and another country, through the medium of one or more other countries, it is called *Indirect Exchange.*

The following table gives the Money of Account of some of the principal places between which and London Exchange business is transacted. The column on the right hand gives the average of the Course of Exchange with London at £1 sterling.

				London gives or receives for £1 sterling.
Amsterdam	20 Stivers	= 1 Florin		12 fl. 3 stv.
Berlin	30 Silv. grosch.	= 1 Thaler		6 thal. 25 s. gr.
*Calcutta	16 Annas	= 1 Rupee		10 rup. 7 an.
Constantinople	40 Paras	= 1 Piastre		109 piastres
Copenhagen	96 Skillings	= 1 Rigsbk. Dlr.		9 r. dalers
*Frankfort	60 Kreuzers	= 1 Florin		12 fl. 10 kr.
Genoa	100 Centisimi	= 1 Lire Nuove		25 l.nuo. 30 c.
Hamburgh	16 Schillings	= 1 Mark		13 mks. 8 sch.
*Lisbon	1000 Reis	= 1 Milreis		4 mi'r. 444 r.
*Madrid	20 Reales	= 1 Hard Dollar		4 h.dol. 16 rls.
Paris	100 Centimes	= 1 Franc		25 fr. 40 c.
*St. Petersburgh	100 Copeks	= 1 Rouble		6 roub. 30 cop.

Ex. 1. Exchange £320. 10s. with Amsterdam at the usual rate.

By reference to the table the usual rate is 12 florins 3 stivers for £1.

And 12 fl. 3 stv. = $12\frac{3}{20}$ fl. = 12·15 fl.

therefore, 12·15 fl. × $320\frac{1}{2}$ = 3894 fl. $1\frac{1}{2}$ stv., *Ans.*

It will generally be found most convenient to reduce that sum (Foreign or English) to the decimal form which is most fractional, and to multiply by the number expressing the remaining sum.

Ex. 2. Exchange £473. 9s. 6d. with Frankfort at the usual rate.

The rate given in the table is 12 florins 10 kreuzers, or $12\frac{1}{6}$ florins.

And, £473. 9s. 6d. = £$473\frac{9}{40}$ = £473·475.

473·475
12$\frac{1}{6}$
5681 700
78 9125
fl. 5760·6125 *Ans.*, 5760 florins 36·75 kreuzers.
60
kr. 36·7500

* The exchange quotations between London and the places marked with an asterisk are not for £1, but as follows:—
Calcutta, 23$\frac{1}{4}$ pence = 1 rupee. Frankfort, 121$\frac{1}{2}$ florins = £10. Lisbon, 54 pence = 1 milreis. Madrid, 50 pence = 1 hard dollar. St. Petersburgh, 38 pence = 1 rouble

Ex. 3. Convert 1000 French francs into the money of St. Petersburgh, at the rate given in the table.

25·4 francs = 6·3 roubles, hence,

1000 × 6·3 ÷ 25·4 = 248 roubles 3 copeks, *Ans.*

Ex. 4. Deduce the rate of exchange between Hamburgh and Berlin from the above table, the quotation with Hamburgh being for 300 marks.

13 marks 8 schillings = 6 thalers 25 silver groschen.

or, $13\frac{1}{3}$ marks = $6\frac{5}{6}$ thalers

therefore, 1 mark = $(6\frac{5}{6} \div 13\frac{1}{3})$ thalers

therefore, 300 marks = $(6\frac{5}{6} \div 13\frac{1}{3})$ thalers × 30

therefore, rate for 300 marks is 151 thalers 25 silv. groschen.

Ex. 5. A merchant in London is indebted to another in Genoa to the amount of 3500 Lire Nuove: if the exchange between London and Genoa and between London and Paris is according to the table given above, and the exchange between Paris and Genoa is 98 francs for 100 Lire Nuove, which will be the most advantageous way for the London merchant to be drawn upon?

98 francs = 100 Lire Nuove, ∴ (98 × 35) francs = 3500 L. Nuove, that is, 3430 francs = 3500 Lire Nuove. Now, if we convert 3430 French francs and 3500 Genoese Lire Nuove respectively into English money, then, since either sum is equivalent to the debt, it follows that the one which requires the least sum of English money in exchange will indicate the more favourable course of exchange.

Paris and London.	Genoa and London.
25·4 francs = £1.	25·3 L. Nuove = £1.
25·4)3430·0(£135·04 sterling.	25·3)3500·0(£138·34 sterling.
254	253
890	970
762	759
1280	2110
1270	2024
1000	860
1016	759
	1010
	1012

Therefore, through Paris is the more advantageous way for the London merchant to be drawn upon, because in that case he would only have to pay £135·04, whereas by direct exchange with Genoa he would have to pay £138·34.

Ex. 6. A merchant buys goods in Paris at 3 francs 30 centimes *per* kilogramme, at what price per lb. should he sell them in London

so as to clear a profit of 20 per cent.? Exchange at 24 fr. 75 centimes for £1, and a kilogramme = 2·2046 lbs.

$$24{\cdot}75 \div 3{\cdot}30 = 7\tfrac{1}{2}$$

therefore, £1 + $7\frac{1}{2}$ = £2 + 15 = 40s. + 15 = 2s. 8d.

hence, $\begin{matrix} 2{\cdot}204 \text{ lbs.} \\ 100 \end{matrix} : \begin{matrix} 1 \text{ lb.} \\ 120 \end{matrix}$:: 2s. 8d. : price required.

whence, price required = $12\frac{4}{5}\frac{4}{5}\frac{4}{1}$d. per lb.

Ex. 54.

Exchange at the rates given in the above table,—

1. £520 with Amsterdam.
2. £307 with Berlin.
3. £67. 10s. with Paris.
4. £325. 5s. with Copenhagen.
5. £824. 4s. with St. Petersbgh.
6. £963. 8s. with Calcutta.
7. £341. 16s. with Genoa.
8. £256. 15s. with Lisbon.
9. £93. 2s. 6d. with Hamburgh.
10. £173. 12s. 6d. with Madrid.
11. £87. 14s. with Frankfort.
12. £437. 17s. with Constantinople.

Exchange into sterling money at—

13. Genoa, 21252 L. Nuove.
14. Amsterdam, 1518 fl. 15 stv.
15. Constantinople, 6921 pi. 20 pa.
16. Copenhagen, 936 skillings.
17. Paris, 839 fr. 47 cent.
18. St. Petersburgh, 340 R. 2 co.
19. Madrid, 1254 reales.
20. Hamburgh, 987 marks.
21. Calcutta, 1012 rup. 7 an.
22. Berlin, 389 thal. 15 s. gr.
23. Lisbon, 469 milr. 953 r.
24. Frankfort, 871 fl. 8 kr.

25. Exchange £121. 3s. 4d. with Amsterdam at 12 fl. 7 stv.

26. Exchange 714 Rigsbank dalers 54 skillings with London at 8 R. dal. 66 skillings per £1 sterling.

27. What sum at Genoa is equivalent to 10 hard dollars at Madrid, the exchange with London being as in the above table?

28. Exchange 300 Hamburgh marks with St. Petersburgh at the rates given in the table.

29. Exchange 3000 French francs with Constantinople.

30. What ought to be the rate of exchange between Lisbon and Paris for 1 milreis, according to the table?

31. Exchange 100 Prussian thalers into French money.

32. How many French francs does Genoa receive for 100 L. Nuove?

33. How many Hamburgh schillings does St. Petersburgh receive for 1 rouble?

34. Exchange 1015 Berlin thalers into French money.

35. In Austria 60 kreuzers = 1 florin, and the rate of exchange between Vienna and London is 12 fl. 16 kreuzers for £1 sterling; what ought to be the rate of exchange between Vienna and Frankfort?

36. A merchant in London is indebted to one in Copenhagen to the amount of 100 Rigsbank dalers; if the rate of exchange be 9¼ R. dal. for £1 sterling, the rate between Amsterdam and London

12 fl. 10 stv. for £1, and the rate between Amsterdam and Copenhagen 135 fl. for 100 R. dalers, which will be the most advantageous way for the London merchant to be drawn upon?

37. If the course of exchange between London and Frankfort be 12½ fl. for £1, between London and Hamburgh 13⅝ marks for £1, and between Hamburgh and Frankfort 88 florins for 100 marks, which will be the most advantageous way for a London merchant to be drawn upon for money due to a Frankfort merchant?

38. Goods are bought in the London market at 1s. 6d. per lb., at what price per kilogramme should they be sold in the French market so as to gain 15 per cent., the rate of exchange being 25¼ francs for £1? (1 kilogramme = 2·2046 lbs.).

39. A French litre = ·22 gallons: what should be the price per gallon in English money corresponding to 1 franc per litre, the exchange being 25 francs for £1 sterling?

40. A last of corn in Amsterdam = 10 3 imperial quarters. If corn be bought in Amsterdam at 150 florins per last, how much per cent. is gained by selling it in the London market at 55s. per quarter, deducting shipping dues at 2s. per quarter? Exchange as in the table.

41. A merchant buys turpentine in Petersburgh at 4 roubles 20 copeks per vedro, 100 vedros being equivalent to 270·49 gallons: at what price per gallon should it be sold in London in order to realize a profit of 20 per cent.? Exchange as in the table.

42. By how much does 216 Rigsbank dalers 18 skillings Copenhagen exceed 110 H. doll. 8 reales Madrid, in English money, the exchange being according to the table?

43. If iron be manufactured in England at £4. 10s. per ton, at what price per 100 kilogrammes should it be sold in Paris so as to clear a profit of 15 per cent., the cost of conveyance being 7s. per ton, and the exchange 25 fr. = £1? (See question 38.)

44. Silk produced in France at 3 fr. 40 cent. per metre is sold in England at 5s. 6d. per yard: the French manufacturer has a profit of 12½ per cent. on the cost of production, and the English buyer pays a duty of 40 per cent. on the cost price. If the rate of exchange be according to the table, how much will the English buyer clear on 2000 metres? (1 metre = 39 37 inches.)

MISCELLANEOUS RULES.

83. *Proportional Division* is a rule for dividing a number into parts which shall have a given ratio to each other. One of the principal applications of the rule occurs in the division of profits between partners in a business *according* to their respective shares. (See Art. 33.)

Ex. 1. Divide the number 56 into two parts which shall be in the ratio of 2 to 5.

$$56 \div (2+5) = 8; \quad \begin{matrix} 8\times 2 = 16 \\ 8\times 5 = 40 \end{matrix} \quad \textit{Ans.}$$

In simple questions like the above the rule is—*Divide the number by the sum of those representing the ratio of the parts, and multiply the quotient separately by those numbers.*

Ex. 2. A, B, and C enter into partnership with capitals of £400, £650, and £350 respectively; their profits amount to £182, what is each one's share?

$$400 + 650 + 350 = 1400$$

$$\frac{182}{1400} \times 400 = \frac{13}{100} \times 400 = £52 \qquad = \text{A's share.}$$

$$\times 650 = £84.\ 10s. = \text{B's} \quad ,,$$

$$\times 350 = £45.\ 10s. = \text{C's} \quad ,,$$

$$\text{proof } £182.\ 0s.$$

Ex. 3. A begins business with a capital of £500, at the end of 6 months B joins him with a capital of £800, and at the end of 8 months from A's commencement C enters the business with a capital of £1300: at the end of the year their profits amount to £464; how should this be divided?

Their respective shares of the profits will be in proportion to their capitals, multiplied by the numbers representing the times; thus,—

$$500 \times 12 = 6000; \quad 800 \times 6 = 4800; \quad 1300 \times 4 = 5200.$$

∴ the proportions are 6000, 4800, and 5200; or, dividing each of these numbers by 400, the proportions will be 15, 12, and 13.

$$£464 \div (15 + 12 + 13) = £11.\ 12s.$$

$$£11.\ 12s. \times 15 = £174 \qquad = \text{A's share.}$$

$$£11.\ 12s. \times 12 = £139.\ 4s. = \text{B's} \quad ,,$$

$$£11.\ 12s. \times 13 = £150.\ 16s. = \text{C's} \quad ,,$$

84. *Equational Forms.* The equality between two arithmetical expressions which are not identical is called an *Equation.*

By the application of one or more of the four fundamental operations to arithmetical equations a variety of questions may be solved.

Ex. 1. A owes B £150 which is due in 6 months, and £210 due in 9 months: in what time may the whole debt be discharged?

The whole debt = £150 + £210 = £360.

Then, 360 × time required = 150 × 6 + 210 × 9

or, 360 × time required = 900 + 1890

or, 360 × time required = 2790

dividing by 360, time required = $7\frac{3}{4}$ months.

In this and similar questions it is assumed that the interest of the aggregate debt for a given time is equal to the sum of the interests of the several debts for the same time. The time when the whole debt may be discharged is called the *equated time.*

Ex. 2. A has to pay B £840 in 12 months: if he pays £560 in 9 months, in what time ought he to pay the remainder?

Remainder of the debt = £840 − 560 = £280.

Then, 560 × 9 + 280 × time required = 840 × 12

or, 5040 + 280 × time required = 10080

subtracting 5040, 280 × time required = 5040

dividing by 280, time required = 18 months.

Ex. 3. A can do a piece of work in 10 days which B can do in 6 days; in what time will they do it together?

A does $\frac{1}{10}$ of the work in 1 day, and B does $\frac{1}{6}$; therefore, A and B together will do $(\frac{1}{10} + \frac{1}{6}) = \frac{4}{15}$ of the work in a day: hence,—

fraction of work done in 1 day × time required = whole work

or, $\frac{4}{15}$ × time required = 1

therefore, time required = $1 \div \frac{4}{15} = 3\frac{3}{4}$ days.

It is always convenient to take unity as representing the whole work.

Ex. 55.

1. Divide the number 132 into two parts, one of which shall be three times as great as the other.

2. Divide £43. 10s. 2½d. among A, B, and C, so that C shall have twice as much as A, and B twice as much as C.

3. A and B begin business with capitals of £700 and £400 respectively; their profits amount to £177. 13s.: what is each one's share?

4. A and B enter into partnership with equal capitals, at the end of 10 months A withdraws from the business, and at the end of 12 months they divide £676. 12s. 0¾d. profits. By how much does B's share exceed A's?

5. A, B, and C have capital in the ratios of 2, 3, and 5 respectively, and jointly it amounts to £5000. If their profits amount to 12½ per *cent.*, what is each one's share?

6. Divide £97. 10s. into three parts, in the ratio of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$.

7. Divide £31. 12s. 6d. into three parts, which shall be in the ratio of ·125 ; 2·4 ; and ·35.

8. A and B enter into partnership ; A puts in £550 for 7 months, and B puts in £750 for 6 months. Their profits amount to £242. 3s. ; what is each one's share ?

9. A, B, and C join together in business ; A's capital is half as much again as B's, and C's is half as much again as A's. If their profits amount to £313. 10s., what is each one's share ?

10. Gunpowder is composed of 78 parts of nitrate of potash, 12 parts of charcoal, and 10 parts of sulphur : how many pounds of each are there in 6 cwt. 3 qrs. 19 lbs. of gunpowder ?

11. A shield composed of gold and silver, in the proportion of 4 parts of gold to 19 of silver, weighs 13 lbs. 8 oz. 9 dwts. ; what is the value of the shield, reckoning gold at £3. 15s. per oz., and silver at 5s. per oz. ?

12. A is indebted to B £300, two-thirds of which are due in 5 months, and the remainder in 8 months : find the time at which the whole debt may be discharged at once.

13. A owes B £1000, due in 6 months, but he pays £600 in 4 months ; in what time should he pay the remainder of the debt ?

14. A owes B three sums of £100 each, due in 3, 4, and 8 months respectively ; find the equated time of payment.

15. A person has debts of £50, £60, and £70, due in 4, 6, and 8 months respectively : find the equated time for the payment of the whole.

16. A can do a piece of work in 3 days which B can do in 4 days ; in what time will they do it together ?

17. A can do a piece of work in 15 days, B in 12 days, and C in 20 days ; in how many days will they do it together ?

18. A, B, and C join their capital in the proportion of 3, 5, and 8 ; they clear £456 profit, being at the rate of 19 per cent. : how much capital did each contribute ?

19. The daily wages of A, B, and C, are in the ratio of 3, $3\frac{1}{2}$, and 4 ; by working the same number of days their united earnings amount to £16. 10s. 9d. : what is each one's share ?

20. A, B, and C can do a piece of work in 8, 9, and 10 days respectively ; if they work together, and each be paid according to merit, what will be their respective shares, the whole sum paid being £45. 7s. 6d. ?

85. *Per Centages.* In the questions we have hitherto given the term *per cent.* has been restricted to 100 units of money,—generally to £100. It may, however, be applied to 100 units of any kind.

Ex. 1. The annual number of deaths in a town containing a population of 3000 is 40: how many is that per cent. ?

The question is,—if the number of deaths in a population of 3000 be 40, what is the number in a population of 100 ?

hence, 3000 : 100 :: 40 : per centage required

whence, per centage = $1\frac{1}{3}$ *Ans.*

Ex. 2. A school contains 150 scholars, of whom 12 per cent. are absent during 3 days a week, and 10 per cent. during the remaining 2 days in the week: what is the per centage of absentees for the whole week.

$$\frac{12\times 3+10\times 2}{\text{whole number of days}}=\frac{56}{5}=11{\cdot}2 \text{ per cent., } \textit{Ans.}$$

Or we may reason thus,—100 : 150 :: 12 : absentees for 3 days; whence, absentees = 18.

Again, 100 : 150 : : 10 : absentees for 2 days; whence, absentees = 15.

Then, $(18\times 3+15\times 2)\div(3+2)=84\div 5=16\frac{4}{5}$ = average number absent for each day in the week.

Finally, 150 : 100 : : $16\frac{4}{5}$: per centage required

whence, absentees for the week = 11·2 per cent.

Ex. 3. There are two schools in a village containing 90 and 120 scholars respectively, but at the end of a month it is found that the former has increased 5·2 per cent., and the latter has decreased 4·75 per cent.: find the increase or decrease per cent. in the two schools together.

100 : 90 :: 5·2 : increase in first school; this = 4·68

100 : 120 : 4·75 : decrease in 2nd school; this = 5·7

whence, decrease in (90 + 120) = 5·7 − 4·68 = 1·02

therefore, (90 + 120) : 100 : : 1·02 : decrease per cent.

whence, we find there is a decrease ·4857 per cent.

86. *Averages.* Some simple questions on averages have already been given under Art. 23, we shall here give one additional example.

Ex. A cattle dealer bought 50 bullocks for 1100 guineas, and six score sheep at £3. 2s. 6d. per head; he gained £3. 15s. 6d. per head on the bullocks, and lost 50 guineas on the whole number of sheep. What was the average loss or gain per head ?

£3. 15s. 6d. × 50 = £188. 15s. = total gain on 50

50 guineas = £ 52. 10s. = total loss on 120

therefore, £136. 5s. = total gain on 170

whence, average gain per head = £136. 5s. ÷ 170 = 17s. $2\frac{5}{17}$d. *Ans.*

Ex. 56.

1. A school consists of 125 scholars, of whom 10 are absent; how many is this per cent. ?

2. The population of a parish is 13450, and the annual number of deaths is 269: find the per centage.

3. Out of 490 members in the House of Commons 315 voted for a measure and 175 against it: what per centage was the majority of the whole ?

4. The population of a town is 6900; one-half are under 30 years of age, and one-third are between 30 and 50: find the per centage above 50 years of age.

5. The population in the Isle of Man in 1821 was 89508; in 1831 it was 103710; in 1841 it was 124040; in 1851 it was 143126: find the increase per cent. for each period of 10 years.

6. In a locomotive engine 9·75 per cent. of the power is lost through friction, and $6\frac{1}{4}$ per cent. owing to the cooling of the steam in its passage to the cylinder: if, without making any allowance for loss of power, the engine be calculated to take a train of 100 tons $22\frac{1}{2}$ miles per hour, how far will it actually take a train of 150 tons in $3\frac{3}{4}$ hrs. ?

SQUARE AND CUBE ROOTS.

87. The *square root* of a number is that number which, multiplied by itself, will produce the given number. The *cube root* of a number is that number which, multiplied by its square, will produce the given number.

Exs. Extract the square roots of 13727025 and ·045369.

$\dot{1}3\dot{7}2\dot{7}0\dot{2}5(3705$ *Ans.*	$\dot{0}\cdot\dot{0}\dot{4}\dot{5}3\dot{6}\dot{9}(\cdot213$ *Ans.*
9	4
$67)472$	$41)53$
469	41
$7405)37025$	$423)1269$
37025	1269

In the first of these examples we begin by placing a point over the unit's figure, and then over every second figure to the left: this divides the number into periods of two figures each, (the left hand period may consist of only one figure, but in this case there are two). We then take the figure whose square is nearest to, and yet does not exceed, the first period 13; this we know to be 3, which we place in a loop to the right; we then write the square of 3, that is 9, under the first period 13 and subtract. We next bring down the second

period 72, and place it by the remainder 4, to form the second dividend; then doubling the first root figure 3 we get 6, which is placed in a loop to the left of the second dividend 472; this 6 must be regarded as 60, which is the trial divisor for 472. Dividing 472 by 60 we find the quotient to be 7, which we place both after the 3 in the root and after the 6 in the loop on the left. The complete divisor is therefore 67, which we multiply by the second root figure 7, and subtract the product 469 from 472. To form the next divisor we double the root figures 37, which gives 74: this 74 is regarded as 740, and is the next trial divisor; the dividend in this case is 370, and since 740 is not contained in 370 we place a 0 both in the root and after the 74. Bringing down the next period 25 we have 37025 as dividend, and 7400 as trial divisor: the quotient is 5 which, as before, we place both in the root and in the loop on the left, making the complete divisor 7405, which multiplied by 5 gives 37025, leaving no remainder when subtracted from the final dividend.

When there is a remainder after bringing down the last period, the operation may be carried to any degree of accuracy by bringing down cyphers and placing a decimal point before the next root figure.

In the second of the above examples we have to extract the square root of a decimal. The operation in this case is the same, excepting the pointing off of the periods, which is from left to right instead of from right to left. If the last period consist of only one figure, a cypher is annexed to make the period complete.

The square root of a vulgar fraction is found by extracting the square roots of the numerator and denominator. The square root of $\frac{2}{4}\frac{5}{9}$, for example is $\frac{5}{7}$.

For the following simple method of extracting the cube root of a number, the author is indebted to the kindness of Mr. Thomas Tate, from whose excellent little book on the "First Principles of Arithmetic" it is taken.

Ex. 3. What is the cube root of 93082856768?

$3 \times 4^2 = 48$
$3 \times 4 \times 5 = 60$
$5^2 = 25$
5425
625
$3 \times 45^2 = 6075$
$3 \times 45 \times 3 = 405$
$3^2 = 9$
611559
4059
$3 \times 453^2 = 615627$
$3 \times 453 \times 2 = 2718$
$2^2 = 4$
61589884

93082856768(4532
64
29082
27125
1957856
1834677
123179768
123179768

"Here we first place a point over the unit's place, and then over every third figure. We take the nearest cube root of 93, which will be 4, with 29 as a remainder. We bring down 082, the figures included by the next point; and take three times the square of the root figure, making 48; this is the trial divisor, then 290 divided by 48 gives 5, which is the next figure of the root. To complete the divisor we take 3 times the preceding root figure multiplied by the new figure of the root, and write the product 60 beneath the 48, taking care to remove the figures a place to the right; in the same manner we write down the square of the new root figure; these added together give 5425 for the true divisor, which multiplied by 5, and subtracted from the subtrahend 29082, gives 1957 as a remainder; and so on to all the other figures of the root. It remains to be shown how the next trial divisor is found without the trouble of squaring 45 and multiplying by 3. Bring down 60 and 25, making 625, and then add the three lines of figures connected by the bracket, and this will give 6075 for the trial divisor; and so on to all the succeeding ones."

The pointing of the periods in a pure decimal number is from left to right, thus,—·031247563. In a mixed decimal number the first point is placed over the unit's figure, and then over every third figure to the right and left.

The cube root of a vulgar fraction is the cube root of its numerator over the cube root of its denominator; thus, the cube root of $\frac{1}{3}\frac{2}{4}\frac{5}{6} = \frac{5}{6}$.

The reasons for the methods employed in the square and cube roots are best understood from the algebraic proofs.

Ex. 57.

Extract the square roots of—

1. 121; 169; 289; 625; 1156; 9409; 11664.
2. 326041; 654481; 10764961; 2649469729.
3. 44894073692416; 1191078144; 7064402500.
4. 452992264209; 7119359165858396732209.
5. ·0529; ·4489; ·103041; ·005041; ·000001.
6. $\frac{1}{4}$; $\frac{2}{6}\frac{5}{4}$; $\frac{3}{7}\frac{2}{1}\frac{5}{6}$; $\frac{2}{1}\frac{0}{0}\frac{0}{0}\frac{0}{1}$; $2\frac{1}{8}$ of $\frac{1}{6}\frac{3}{1}$ of $20\frac{4}{1}\frac{1}{3}$ of $\frac{9}{1}\frac{2}{7}\frac{1}{6}$.

Find the cube roots of—

7. 729; 6859; 274625; 30371328; 371694959.
8. 70189453125; 220677361507; 513729944729000.
9. ·125; ·000343; ·032768; ·000242970624.
10. $\frac{1}{8}$; $\frac{3}{6}\frac{7}{4}\frac{7}{3}$; $\frac{2}{2}\frac{1}{1}\frac{8}{0}\frac{8}{7}$; $1\frac{7}{8}$ of $4\frac{1}{8}$ of $\frac{3}{5}\frac{7}{5}$ of $8\frac{8}{1}\frac{5}{1}$ of $\frac{1}{1}\frac{1}{0}\frac{1}{3}\frac{1}{2}$.
11. Find the square root of 34·1 correct to five decimal places.
12. Find the cube root of 6314 correct to four places of decimals.
13. A square field contains 75 ac. 20 po. 4 sq. yds., how many yards are there in the length of its side?
14. A cubical block of marble weighs 129 tons 3 cwt. 3 qrs. 26 lbs. 4 oz.; find the length of its edge, supposing a cubic foot of marble to weigh 3 cwt. 3 qrs. 12 lbs.

Miscellaneous Exercises.

1.

1. Multiply 643759 by 759468; and divide 5486206094 by 8147.
2. Multiply £369. 17s. 5¾d. by 59.
3. How many times is £17. 13s. 2¼d. contained in £6692. 18s. 0¾d.?
4. Reduce 7 tons 13 cwt. 1 qr. 23 lbs. 13 oz. to ounces.
5. Find the G. C. M. of 35802 and 55055.
6. If 9 men can do a piece of work in 8 days, in what time will 12 men do it?
7. Find by Practice the value of 6073 articles at £2. 16s. 8d. each.
8. A gallon of brandy costs as much as a gallon and a half of wine, and a gallon of wine and brandy mixed in equal proportions costs 27s. 6d.: find the price of brandy and of wine per gallon.

2.

1. Twenty-nine bags, each containing the same number of farthings, are worth £302. 1s. 8d.; how many farthings are there in each bag?
2. A grocer buys a cask of sugar weighing 7 cwt. 3 qrs. for £17. 3s. 7d., and retails it at 5½d. per lb.; what is his total profit?
3. A boat's crew can pull 10 miles an hour with the tide, and 5 miles an hour against it; what is the rate of the tide per hour?
4. Find the L. C. M. of 12, 28, 16, 18, 30, 35, 21, and 40?
5. Find by Practice the value of 2 tons 11 cwt. 3 qrs. 19 lbs. at 35s. per cwt.
6. A gentleman's income is £1200 a year, and he saves £536. 18s. 4d.: what is his average daily expenditure?
7. Divide £1727. 4 fl. 9 c. 9 m. by 57.
8. Add together $3\frac{1}{4}$, $4\frac{1}{3}$, $2\frac{1}{8}$, $\frac{5}{8}$, $1\frac{5}{12}$, and $4\frac{9}{16}$.

3.

1. Add together £83. 9s. 4d., £216. 17s. 11½d., £57. 8s. 6¾d., £10. 19s. 8d., £398. 15s. 9d., £209. 19s. 0½d., £63. 8s. 10½d., and divide the sum by 392.
2. Divide £4. 13s. 5¾d. between two persons so that for every shilling one receives the other shall receive half-a-crown.
3. A gold chain weighing 3 oz. 17 dwt. is composed of 15 parts of fine gold, and 9 parts of alloy; what is the value of the gold at £4. 3s. 4d. per oz.?
4. Express £25. 0s. 6d. in the decimal coinage.
5. If a tradesman gain 2¼d. on half-a-crown what is his profit per cent.?
6. Multiply $2\frac{5}{11}$ of $1\frac{5}{8}$ of $\frac{13}{87}$ by $5\frac{5}{8}$ of $\frac{19}{119}$ of $9\frac{11}{13}$ of $\frac{11}{15}$.
7. If 3 cwt. 1 qr. 17 lbs. of goods cost £16. 5s. 5½d., what is the cost of 2 tons 17 cwt. 23 lbs.?
8. *Find the* simple interest on £168 for $3\frac{3}{4}$ yrs. at $5\frac{1}{4}$ p.c.

4.

1. Add together £89. 17s. 6½d., £73. 8s. 9d., £263. 15s. 4¾d., £68. 8s. 8d., £9. 19s. 11½d., £56. 4s. 0¾d., £516. 6s. 10d., £870. 9s. 5d., £8. 7s. 8½d., £93. 16s. 1d., £478. 10s. 11d., £6. 8s. 9½d., £149. 4s. 6½d., £31. 2s. 4d., £640. 18s. 11½d., £18. 18s. 5d., £93. 9s., £6. 14s. 7¾d., £9. 7s. 8d., £448. 5s. 4d., and £87. 19s. 3½d.

2. A grocer buys 4 chests of tea, weighing 58 lbs. each, for £36. 7s. 3d., and a hogshead of sugar, containing three-quarters of a ton, at £2. 14s. 10d. per cwt.; he sells the tea at 3s. 8d. per lb., and the sugar at 5½d. per lb. What profit will he have cleared, after selling the whole of the tea and one-half the sugar?

3. Divide 6·1314 by ·00384.

4. A rectangular garden is twice as long as it is wide, and contains 4608 square yards; what is the length of the fence inclosing it?

5. Add together $7\frac{1}{4}$, $\frac{2}{3}$ of $3\frac{1}{3}$ of $4\frac{1}{5}$, and $\frac{\cdot 063}{\frac{3}{8} \text{ of } \frac{7}{12}}$

6. Eggs are bought at 9d. a dozen, and sold at 15 for a shilling; what is the profit per cent.?

7. Find the mercantile discount on £350, due 8 months hence, at 7 per cent. per annum.

8. Tea is bought at 5s. per lb., and sold at £·325. per lb.; what is the profit per cent.?

5.

1. Multiply 5791030046 by 794.
2. Divide 49372527710935 by 80407.
3. Multiply £6. 15s. 8¾d. by 7561.
4. Divide £66527. 18s. 1½d. by 3705.
5. Divide £13522. 14s. 1¾d. by £37. 13s. 4¼d.
6. Multiply 7 oz. 17 dwt. 19 grs. by 647.
7. What is the greatest number that will divide both 8502 and 61126 without remainder?
8. Find the L. C. M. of 20, 24, 28, 30, 32, 36, and 42.

6.

1. Multiply 6429738579 by 6957.
2. Divide 709609620160326 by 8374.
3. Multiply £19. 17s. 9½d. by 431.
4. Multiply 3 cwt. 19 lbs. by 353.
5. Divide £15133. 2s. 5¾d. by 247.
6. Divide 2982 tons 3 cwt. 9 lbs. by 7 cwt. 1 qr. 13 lbs.
7. A merchant buys 2 tons 13 cwt. 19 lbs. of sugar for £97. 10s., and sells it at 2 guineas per cwt. What does he clear on the whole?
8. If 12 men can mow 18 acres of grass in 2 days, how many acres will 17 men mow in 11 days?

7.

1. By what number must 83130854890488 be divided so as to give 9458 for the quotient?
2. If £30650. 7s. 6¾d. be divided equally among 93 persons, what sum will each receive?
3. Divide 1272 m. 1 fur. 20 po. 3 yds. 1 ft. 6 in. by 1 m. 3 fur. 31 po. 4 yds.
4. A tradesman buys goods at £1. 11s. 9½d. per cwt., and pays in addition 3s. 9½d. per cwt. for carriage: if he retails the goods at 4¼d. per lb., what profit will he realise on 7 cwt. of goods?
5. Find by Practice the value of 2417 articles at £1. 14s. 6d. each.
6. Find by Practice the value of 2 tons 3 qrs. 17 lbs., at 11s. 8d. per cwt.
7. If 11 cwt. 2 qrs. 13 lbs. of iron cost £4. 1s. 3¾d., what ought to be the cost of 3 tons 3 qrs.?
8. Reduce $8\frac{1}{4}$ of $\frac{5}{7}$ of $9\frac{1}{2}$ of $\frac{8}{11}$ of $\frac{4}{15}$ to a simple fraction.

8.

1. Divide 544360788704200 by 84136.
2. Prize money to the amount of £2337. 17s. 9¾d. is to be divided equally among a certain number of men, each receiving £13. 13s. 5½d.; find the number of men.
3. Find the L. C. M. of 85, 117, 51, 39, 136, 34, and 57.
4. Reduce $8\frac{1}{4}$ of $5\frac{1}{2}$ of $1\frac{1}{7}$ of $3\frac{1}{7}$ of $\frac{3}{15}$ to a simple fraction.
5. Add together $4\frac{3}{4} + \frac{5}{8}$ of $3\frac{3}{4}$, and $2\frac{7}{10}$.
6. From $\frac{5}{7}$ of $4\frac{3}{5}$ of $1\frac{8}{19}$ take $\frac{3}{11}$ of $\frac{4}{5}$ of $4\frac{1}{2}$.
7. Find the simple interest on £960 for $2\frac{1}{2}$ years at $3\frac{1}{4}$ per cent. per annum.
8. A grocer cleared £70 profit by selling sugar at 1½d. per pound more than it cost him; how much did he sell.

9.

1. The circumference of a coach-wheel is 13 ft. 6 in., and it makes 2640 revolutions between two towns; what is the distance between the towns?
2. Take 1000 guineas from 5173 crowns.
3. Divide 80 lbs. 3 oz. 7 drs. 19 grs. by 2 oz. 5 drs. 2 scr. 17 grs.
4. A nugget is found to weigh 32 lbs. 5 oz. 16 dwt. 6 grs., of which five-sevenths is refuse and the remainder pure gold. Find its value, the price of gold being £3. 17s. 6d. per oz.
5. Divide $6\frac{1}{4}$ of $\frac{8}{11}$ of $\frac{1}{11}$ by $1\frac{2}{7}$ of $\frac{1}{13}$ of $\frac{1}{3}$.
6. A person receives $3\frac{1}{2}$ per cent. interest by investing money in 3 per cent. stock; what is the price of the stock?

7. A merchant buys goods at £3. 7s. per cwt., and sells them at 11d. per lb.; what is his profit per cent.?

8. Simplify $\frac{3\frac{3}{4}-1\frac{1}{2}}{3\frac{3}{4}+2\frac{2}{3}}$ of $9\frac{1}{3}$ of $\frac{1\frac{2}{3}}{2\frac{1}{2}}$.

10.

1. Divide £9826. 10s. 6¾d. by £53. 13s. 11½d.

2. Multiply 7 oz. 13 dwt. 19 grs. by 239.

3. In a school containing 84 scholars, eleven are absent on Monday, nine on Tuesday, six on Wednesday, four on Thursday, and five on Friday; what is the percentage of absentees for the whole week?

4. Simplify $8\frac{1}{4}$ of $\frac{3}{7}$ of $\cdot\dot{6}\dot{3}$.

5. A tradesman's profits amount to 12½ per cent. on his outlay; what ought to be the selling price of an article which cost £4. 5s.?

6. Divide ·21 by ·0017.

7. Find the expense of paving a rectangular court-yard 33 ft. long and 25 ft. wide, at 2s. 2¼d. per square yard.

8. Find the simple interest on £875. for 2½ years at 5½ per cent.

11.

1. Add together 89635, 47239, 6048, 76314, 976352, 8763, 94663, 843596, 349, 8973, 68, and 942768, and divide the sum by 749.

2. Add together £29. 7s. 1½d., £341. 0s. 9½d., £67. 19s. 0¾d., £98. 4s. 11d., £974. 18s. 6½d., £65. 0s. 3½d., £8. 14s. 10¼d. £59. 17s. 4½d., and £843. 10s. 5½d., and divide the sum by 879.

3. If 15 men can mow 7 acres of grass in 3½ days, in how many days will 9 men mow 12 acres.

4. By selling an article at 10s. a tradesman gains 20 per cent.; find the cost price?

5. Divide $\frac{3}{11}$ of $9\frac{1}{2}$ by $4\frac{3}{4}$ of $\frac{9}{44}$.

6. Find the simple interest on £871. 13s. 4d., for 4½ years, at 3⅛ per cent.

7. What sum must be invested in the 3 per cents. at 91 to realise an income of £136. 10s.

8. Tea at 4s. 6d. is mixed in equal quantities with tea at 5s. 6d., and the mixture is sold at a profit of 15 per cent.; find the selling price per lb.

12.

1. Multiply £93. 17s. 8½d. by 769.

2. A boat's crew can pull 9 miles an hour down a river; but, owing to the stream, they can only pull 4 miles per hour up the river. In what time will they pull 3 miles down the river and back again?

3. From $7\frac{1}{2}$ of $1\frac{3}{5}$ take $3\frac{1}{8}$ of $\frac{2}{15}$.

4. Two market women meet, and one gives the other 20 apples for 6 eggs; the one who gives the apples loses 5 per cent. by the exchange. If eggs are selling at 10 for a shilling, what are apples a dozen?

5. Reduce $\frac{2}{3}$ of $1\frac{1}{2}$ of 1s. 7d. to the fraction of $2\frac{1}{4}$ of $3\frac{1}{3}$ of 2s. 11d.

6. A can do a piece of work in 12 hrs. which B can do in 10 hrs.; they work together for 4 hrs., when A leaves. In what time will B finish the work?

7. Find the compound interest on £1200, for 3 years, at 5 per cent. per annum.

8. What decimal of a guinea is $\frac{\frac{3}{4}+\frac{7}{15}}{\frac{3}{5}-\frac{1}{6}}$ of £1?

13.

1. A coal merchant clears £100 profit by selling 750 tons of coal; what was his profit per ton?

2. A horse-dealer bought 97 horses; for the first he gave £20, for the second £21, for the third £22, &c. What did he give for the whole?

3. A, B, and C run a race; A can give B just 1 yard in 100 without losing, and C can give A just 1 yard in 121 without losing. If they all start from the same mark, what will be the distance between B and C, when A has run 150 yards?

4. Express $\cdot 03\dot{5}71428\dot{}$ as a vulgar fraction.

5. Add together $3\frac{3}{4}$, $4\frac{1}{8}$, and $5\frac{3}{7}$.

6. If eggs are bought at 13d. a dozen, and sold at 11 for a shilling, how many shillings' worth must be sold so as to gain a penny?

7. Divide $\frac{3\frac{1}{2}}{\frac{7}{9}}$ by $1\frac{1}{4}$ of $3\frac{5}{7}$.

8. A sells goods to B at a profit of 25 per cent., and B sells the goods to C for £50, thereby losing 4 per cent.; what did the goods cost A?

14.

1. Find by Practice the value of 2541 articles at £3. 19s. $2\frac{1}{2}$d. each.

2. Find the value of 3lbs. 11 oz. 16 dwts. 6 grs. at 5s. per oz.

3. Find the continued product of ·304, 4·02, and ·005.

4. Add together $\frac{3}{5}$, $\frac{1}{7}$, and $2\frac{1}{3}$ of $\frac{3}{11}$.

5. What percentage of profit is obtained by selling a pound's worth of goods for a guinea?

6. A horse and gig were bought for £120, but the horse cost half as much again as the gig; the horse was sold at a profit of 10 per cent., and the gig at a loss of 10 per cent. What was the profit per cent. on the whole outlay?

7. Find the simple interest on £855 for 3 years at $7\frac{1}{2}$ per cent.

8. Find the compound interest on £125 for 2 years at 8 per cent. *per annum*.

15.

1. A bankrupt pays a dividend of 13s. 6d. in the pound; how much will be lost on a debt of £2125?

2. The interest on a given sum of money for 4 years is £30; in what time will half the sum produce double the interest at one-third the rate?

3. Divide $\frac{6\frac{3}{5}+7\frac{1}{4}}{9\frac{1}{7}-4\frac{1}{6}}$ by $13\frac{3}{16}$ of $\frac{2\frac{1}{2}}{4\frac{4}{5}}$.

4. Multiply the cube root of 531441 by its square root.

5. What will be the expense of painting the outside of a cubical box whose edge is $3\frac{1}{2}$ feet, at $3\frac{1}{2}$d. per square foot.

6. If the eightpenny loaf weighs 6 lbs. when wheat is 48s. per quarter, what ought to be the price of wheat when the sixpenny loaf weighs 4 lbs.?

7. Express $\frac{5\frac{3}{5}-2\frac{2}{5}}{4\frac{4}{11}\text{ of }\frac{3}{16}} \times \dot{0}\dot{9} + \dot{0}\dot{2}$ as a vulgar fraction.

8. Reduce ·31 of 5s. 6d. to the decimal of $1\frac{1}{14}$ of 7s. 4d.

16.

1. Three chests of tea weighing 73 lbs. each are bought at 3s. 2d., 3s. 7d., and 3s. 9d., respectively. If the whole be mixed together and sold at 4s. per lb., what will be the total profit?

2. Multiply $5\frac{1}{4}$ of $\frac{4}{7}$ by ·0037.

3. Divide $\frac{2\frac{1}{5}}{1\frac{7}{15}}$ by $\frac{\cdot063}{2\frac{1}{4}}$

4. A tradesman's returns amount to £1. 2fl. 5c. for every guinea he invests in his business; what is his profit per cent.?

5. Find the cost of $6\frac{3}{4}$ dozen slates at £1. 12s. per gross.

6. If the price of gold be £3. 17s. 10d. per oz., and that of platinum be £2. 8s. $7\frac{1}{2}$d. per oz., what weight of platinum should be given in exchange for 1 lb. 3 oz. of gold?

7. Divide £31. 7s. $2\frac{1}{2}$d. between A, B, and C, so that A may have half as much, and B twice as much as C.

8. The rent of a farm containing 362 ac. 3 ro. is £1197. 1s. 6d., but on the expiry of the lease, 143 acres of this are let to another tenant. What ought to be the future rent?

17.

1. A can do a piece of work in 8 days, B in 9 days, and C in 12 days; if they all work together for 2 days, in what time will C alone finish the remainder of the work?

2. Simplify $\frac{7\frac{1}{3}-2\frac{1}{3}}{7\frac{1}{3}+1\frac{1}{3}} - 10\frac{1}{4} + 33\frac{1}{3}\frac{3}{4}$.

3. A wine merchant has three kinds of wine, marked X, Y, and Z. A mixture consisting of equal parts of X and Y he sells at 11s. per gallon; a mixture of equal parts of Y and Z he sells at 12s. 6d. per gallon; and a mixture of equal parts of X and Z he sells at 13s. 6d. per gallon. Find the price of each per gallon.

4. What is the present worth of £350, due in half a year, at $4\frac{1}{2}$ per cent. per annum?

5. If a pipe can discharge 7 cubic feet of water in 4 minutes, how long will it take to fill a cistern whose inside dimensions are 8 ft. 6 in. long, 6 ft. wide, and 3 ft. 6 in. deep?

6. Extract the fourth root of 104060401.

7. A person invests £2145 in $3\frac{1}{2}$ per cent. stock at $97\frac{1}{2}$, and afterwards, when the stock has risen to *par*, he sells out, and invests his money in $3\frac{1}{2}$ per cent. stock at 88. Find the increase in his income.

8. Simplify $6\frac{1}{4} + 4\frac{1}{6} - \frac{1}{3}$ of $6\frac{1}{4} \div \frac{3}{7}$ of $1\frac{1}{4}$ of $2\frac{1}{3}$.

18.

1. Find the value of 6 cwt. 2 qrs. 25 lbs. at 15s. 9d. per qr.

2. What part of a guinea is $3\frac{1}{3}$ of $\frac{4}{5}$ of 2s. $7\frac{1}{2}$d.?

3. What will be the expense of papering a room 19 feet long, 17 feet wide, and 13 feet high, with paper $\frac{7}{8}$ of a yard wide, at $5\frac{1}{4}$d. per yard?

4. A gentleman's income, after paying a property tax of 1s. 2d. in the pound, and a poor's rate of $9\frac{1}{4}$d. in the pound, is £1533. 10s. 10d. What is his gross income?

5. A tradesman will lose 8 per cent. by selling an article for 1s. 11d.; how much per cent. will he gain by selling it for half-a-crown?

6. A, B, and C enter into partnership; A puts in £800 capital for 7 months, B puts in £700 for 9 months, and C £500 for 12 months. The total profit is £358; what is each one's share?

7. Simplify $\frac{1\frac{1}{15} \text{ of } \frac{7}{18}}{5\frac{1}{4} \times 4\frac{4}{6} \div 1\frac{1}{3}} \times 5\frac{2}{5} + 1\frac{9}{10}$.

8. What interest is obtained by investing money in 3 per cent. stock at $93\frac{3}{8}$, reckoning $\frac{1}{8}$ for brokerage?

19.

1. What sum of money will amount to 150 guineas in $3\frac{3}{4}$ years at $3\frac{1}{5}$ per cent., simple interest?

2. At what price must tea, which cost 3s. 9d. per lb., be sold, so as to gain 15 per cent.?

3. From $2\frac{1}{5}$ of $3\frac{1}{3}$ of $\frac{10}{11}$ take $1\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{3}{7}$.

4. If 38 tons 15 cwt. 2 qrs 19 lbs. cost £156. 5s., how much should be given for 3 cwt. 2 qrs. 25 lbs.?

5. Add together $2\frac{1}{2}$ of $1\frac{1}{5}$ of $2\frac{1}{4}$ of 1s. 8d. and $\cdot 3\dot{7}\dot{8}$ of 8s. 3d.

6. How many hurdles each, 7 feet long, will be required to enclose a square piece of ground whose area is 2 ac. 3 ro. 28 po. 4 sq. yds.?

7. What will be the expense of making a gravel walk, 6 feet wide, around a square garden whose area is the tenth part of an acre, at 1s. 5d. per square yard?

8. A tradesman bought £100 worth of goods, and sold one-fourth of the goods at a loss of 5 per cent.; one-third at a profit of 4 per cent.; and the remainder at a profit of 6 per cent. What was his total profit?

20.

1. A bankrupt's debts amount to £3500, and his assets to £1480. 4s. 2d. How much in the pound can he pay?

2. What is the rent of a farm containing 353 ac. 2 ro. 37 po., at an average of £3. 10s. per acre?

3. A cistern can be filled in an hour by one pipe, or in 40 minutes by another pipe; it can also be emptied in half-an-hour by a third pipe. In what time will it be filled if they are all left open together?

4. Multiply $\cdot 5\dot{4}$ by $3\cdot 8\dot{5}7142\dot{8}$.

5. Divide $\cdot 7\dot{3}\dot{9}$ by $\cdot 6\dot{7}$.

6. If 9 men can build a wall 63 yards long, 8 feet high, and 14 in. thick, in 4 days of 10 hours each, how many hours a day must 11 men work, in order to build a wall 98 yards long, 5 ft. 6 in. high, and 9 in. thick, in 3 days?

7. In what time will £337. 10s. amount to £364. 10s. at $3\frac{1}{2}$ per cent., simple interest?

8. A manufactures an article which he sells to B; B sells the article to C, who, in his turn, sells the article for £207. 7s. $2\frac{1}{2}$d. If A, B, and C each cleared a profit of 20 per cent. on the sale of the article, what did it cost A to make it?

21.

1. If A can hit a mark three times out of five, B four times out of seven, and C eight times out of thirteen, which is the most likely to win, if each has only a single shot?

2. Simplify $\cdot 5\dot{3} \div 3\frac{1}{3} \times \cdot 3 \times \cdot 0\dot{1}\dot{3} \times 41\frac{1}{4} \div \frac{11}{30}$, and give the result in a decimal form.

3. What sum of money must be invested at $6\frac{1}{2}$ per cent. so as to produce an annual income of £286. 13s.?

4. The section of the bore of a pipe is equal to a square inch; at what rate per second must water flow through it in order to fill a cistern 7 ft. 6 in. long, 3 ft. 6 in. wide, and 2 ft. 8 in. deep, in 14 minutes?

5. Exchange 1043 rupees 7 annas into sterling money. (*See* p. 135.)

6. Find the amount of £1000 for 4 years at 10 per cent., compound interest.

7. A and B start on a holiday tour for 12 days. A's expenses amount to 10s. the first day, 11s. the second, 12s. the third, and so on, continuing to increase by 1s. each succeeding day. In like manner, B's expenses for the first day amount to 14s., and continue to increase 6d. for each succeeding day. What is the *average* of their joint expenses per day?

8. Extract the cube root of 50246268435243343.

22.

1. A tradesman proposes to reduce the price of his goods $3\frac{1}{2}$ per cent.; what must be the selling price of an article marked at 8s. 9d.?

2. If A gives B 10 per cent of the money in his pocket, and B gives A 12 per cent. of his money, A will have 6d. less than he had at first; how much had each at first?

3. Find, by Practice, the rent of a field containing 17 ac. 1 ro. 21 po. $13\frac{3}{4}$ sq. yds., at £4. 2s. 6d. per acre.

4. Simplify $\frac{2\frac{5}{6} \text{ of } \frac{8}{11} \text{ of } 1\frac{3}{12}}{1\frac{5}{9} \text{ of } 1\frac{7}{10} \text{ of } \frac{4}{11}} + \frac{1\frac{3}{15} \text{ of } 1\frac{1}{14} \text{ of } \frac{2}{15}}{\frac{3}{4} - \frac{2}{5}}$

5. A square garden containing 3 ro. 11 po. $6\frac{1}{4}$ sq. yds. has a brick wall 8 feet high and 9 in. thick running all round it. The wall is to be pointed both inside and outside, at $2\frac{1}{2}$d. per square yard. How much more will it cost to point the outside of the wall than the inside?

6. A father leaves to his three sons £4,900, to be divided in the proportion of 3, 5, and 7. Find the share of each, after deducting a legacy duty of $12\frac{1}{2}$ per cent.

7. A can do a piece of work in 6 days, which A and B together can do in 4 days. If A's wages are 8s. per day, what should B's wages be, at the same rate?

8. What is the interest of a million pounds for one day at £6. 1s. 8d. per cent. per annum?

23.

1. A town whose population is 22575 increases to 24381; find the increase per cent.

2. The cube root of a number is $\cdot\dot{8}3$. Find the number.

3. A solid cast iron column, 1 foot square and 10 feet high, weighs 2 tons 14 cwt. 2 qrs. What would have been its weight if it had been cast with a hole 6 inches square through the length of it?

4. A person invests £1474 in $3\frac{1}{2}$ per cent. stock at $92\frac{1}{2}$, and sells out when the stock has risen to *par*, and invests his money in railway shares paying $3\frac{1}{2}$ per cent. at 96. Find the increase in his income.

5. What part of a guinea is $\frac{1}{4}$ of $\cdot 72916\dot{6}$ of £1. 7s.?

6. If the 4 lb. loaf cost 5d., how much per cent. has the price of bread advanced when the 6 lb. loaf costs 9d.?

7. How many fruit trees, each 8 yards apart, can be planted in a square orchard containing 2 ac. 37 po. $16\frac{3}{4}$ sq. yds., so that no tree may be within 8 yards of the fence?

8. Find the true discount on a bill for £560, due 8 months hence, *at* $7\frac{1}{2}$ *per* cent. per annum.

24.

1. Find by Practice the value of 347 articles at £2. 17s. 10d. each.
2. A grocer sells tea at 3s. 10d. per lb. which he buys at 3s. 4d.; what is his profit per cent.?
3. Find the cost of 3 tons 13 cwt. 21 lbs. at 11s. 9d. per quarter.
4. A can do a piece of work in 6 days which B can do in 10 days; in how many days will they do it together?
5. Find the area of a square field whose side measures 231 yds.
6. Multiply 5·631 by ·043, and divide ·02028 by ·624.
7. If 6 men can finish a piece of work in 5 days by working 12 hrs. a day, how many men will finish it in 8 days working 9 hrs. a day?
8. Find the compound interest on £260 for 2 years at 4 p. c.

25.

1. Multiply £679. 17s. 5¾d. by 83; and divide 191 tons 7 cwt. 2 qrs. 25 lbs. by 479.
2. How many times is £6. 15s. 9½d. contained in £257258. 11s. 6d.?
3. Find the G. C. M. of 138138 and 3433716.
4. Add together $3\frac{3}{7}$, $4\frac{1}{5}$ of $2\frac{1}{7}$ of $\frac{5}{1\,8}$, and $\frac{3\,9}{1\,8\,2}$.
5. If 57 yards of cloth are bought for £44. 17s. 9d., what is the cost of 197 yards?
6. Multiply ·00221 by ·0437 and divide the product by 3·23.
7. Sugar is bought at 2 guineas per cwt. and sold at the rate of £3. 7s. 1d. for 1 cwt. 1 qr.; what is the profit per lb.?
8. How many square yards are there in the surface of a cube whose edge measures 3 ft. 9 in.?

26.

1. Divide 40 guineas into an equal number of guineas, pounds, half-guineas, half-crowns, and florins.
2. Divide $3\frac{1}{8} + \frac{3}{5}$ of $\frac{1}{16} + 7\frac{1}{3}$ of $\frac{6}{11}$ by $5\frac{1}{10}$.
3. Express $\cdot\dot{1}8\dot{5}$ as a vulgar fraction in its lowest terms.
4. Extract the square and cube roots of 244140625.
5. At what rate per cent. will £10 become 12 guineas in 4 years at simple interest?
6. How much will it cost to paper the walls of a room 23 ft. 6 in. long, 16 ft. 6 in. wide, and 12 ft. high, with paper three quarters of a yard wide, at 6¾d. per yard?
7. What quantity of stock at $90\frac{5}{8}$ can be bought for £508. 4s., reckoning $\frac{1}{8}$ for brokerage?
8. A farmer sold 20 bushels of oats at 4s. 1½d. per bushel, 16 bushels of wheat at 6s. 6d. per bushel, and 25 bushels of barley at 4s. 9d. per bushel. What was the average price per bushel for the whole?

27.

1. A dealer buys 100 sheep for 250 guineas and sells two-fifths of them at £2. 15s. each; at what price per head must he sell the remainder so as to gain 10 per cent on the whole?

2. Divide the continued product of $3{\cdot}4 \times {\cdot}76 \times {\cdot}051$ by ·008.

3. Express $\frac{5}{11}$ of $6\frac{2}{7}$ of $\cdot\dot{8}\dot{3}$ as a decimal.

4. A person invests 1000 guineas in railway shares at 75, and when the shares have risen to *par*, and are paying $4\frac{1}{2}$ per cent., he transfers his capital to the $3\frac{1}{4}$ per cent. consols at $81\frac{1}{4}$: find the loss or gain in his annual income.

5. A merchant bought £500 worth of sugar and £300 worth of tea; he sold the sugar at a profit of 3 per cent., and the tea at a loss of 2 per cent. What was his profit per cent. on his whole outlay.

6. Divide the cube root of 10·077696 by the square root of ·00005184.

7. A room twice as long as it is wide costs £13. 1s. 4d. to carpet it with carpet $\frac{7}{8}$ of a yard wide, at 5s. 3d. per yard: find the length of the room.

8. What number bears the same ratio to ·012 as $\frac{3}{5}$ of $4\frac{1}{7}$ of ·375 does to $9\frac{3}{14}$ of 2·97 of $1\frac{1}{45}$ of $\cdot00\dot{4}\dot{5}$?

28.

1. If £1283. 8 fl. 5 c. be divided equally among 317 persons, what sum will each receive in current coin?

2. If 14 cwt. 3 qrs. 1 lb. cost £105. 9s., what is the cost of 1 cwt. 3 qrs. 7 lbs.?

3. A map is 4 ft. 6 in. long by 3 ft. 3 in. broad, and $\frac{4}{5}$ of its surface represent water; how many square inches represent land?

4. Multiply $\frac{2}{7} + \frac{3}{4}$ of $5\frac{3}{5}$ of $3\frac{1}{3}$ by $\frac{7}{8}$ of $\frac{7}{17}$.

5. How much more per cent. interest will a person receive by transferring his money from the $3\frac{1}{2}$ per cents. at 98 to the 3 per cents. at 80?

6. A gives $\frac{1}{3}$ of his money to B, $\frac{1}{4}$ of the remainder to C, $\frac{1}{5}$ of what still remains to D, and then he has 3 guineas left: how much had he at first?

7. Find the true discount on a bill for £97. 17s. due in 219 days at 3 per cent.

8. If a pocket of hops weighing $3\frac{3}{4}$ cwt. be sold for £82. 10s., and an acre of ground produce 5 cwt. 2 qrs. 14 lbs., what sum should be realized from the produce of a field containing 7 ac. 1 ro. 24 po.?

ANSWERS.

Ex. 1.—*Page* 4.

1. 14	5. 429	9. 72035	13. 1000100	17. 9090900
2. 18	6. 294	10. 426799	14. 4070080	18. 7700500
3. 81	7. 8005	11. 50005	15. 6006000	19. 60000002
4. 97	8. 5080	12. 300301	16. 800088	20. 2405601

Second Course.—Page 5.

1. 360000000	4. 54070603090	7. 6002000000001
2. 507063014	5. 200001840329	8. 907402000800800
3. 7000009000	6. 30000040050000	9. 803733958
	10. 4495748	

The following numbers expressed in words are the answers to (18) and (19):—18. 471; 417; 741; 714; 147; 174. 19. 6800; 6080; 6008; 8600; 8060; 8006.

Ex. 2.—*Page* 7.

1. 18	15. 31	29. 342	43. 3610	57. 46360	71. 458928			
2. 18	16. 33	30. 371	44. 5574	58. 40665	72. 529928			
3. 20	17. 37	31. 266	45. 4301	59. 35112	73. 479041			
4. 22	18. 33	32. 246	46. 5407	60. 34893	74. 222437			
5. 23	19. 33	33. 343	47. 37040	61. 20356	75. 82758			
6. 22	20. 28	34. 193	48. 28645	62. 59824	76. 645706			
7. 22	21. 33	35. 222	49. 28090	63. 541112	77. 6916369			
8. 23	22. 29	36. 474	50. 36577	64. 532161	78. 7725710			
9. 26	23. 42	37. 4884	51. 33216	65. 493363	79. 87275612			
10. 24	24. 46	38. 4921	52. 23078	66. 491546	80. 915092			
11. 26	25. 284	39. 4204	53. 38519	67. 548870	81. 93903			
12. 28	26. 299	40. 4241	54. 16531	68. 462550	82. 1039946			
13. 35	27. 307	41. 1869	55. 41013	69. 540260	83. 1005266			
14. 31	28. 299	42. 4787	56. 47906	70. 194123	84. 1103631			

Second Course.—Page 9.

1. 114413	8. 57153374	15. 7247490	22. 5175
2. 71139	9. 5086164	16. 62640903	23. 10721
3. 24835 *miles*	10. 12368416	17. 17590751518082	24. 259729
4. £1244135	11. 61867377	18. 76081922	25. 60522
5. 1449725	12. 2024077	19. 277	26. 119
6. 1805769	13. 438	20. 501	27. 14684
7. 117311365	14. 8542231	21. 65161469	28. 67; 72; 1848

Ex. 3.—*Page* 12.

1. 123224	9. 333333	17. 656754	25. 315071
2. 244152	10. 41515	18. 504545	26. 509997
3. 310441	11. 123403	19. 212690	27. 305337
4. 411003	12. 34253	20. 185948	28. 312949
5. 31302	13. 835442	21. 383037	29. 88080
6. 300033	14. 237196	22. 299169	30. 360486
7. 53213	15. 2346	23. 197254	31. 219831
8. 403003	16. 790542	24. 91998	32. 248906

33. 232592 37. 58195937 38. 19629980 43. 425178
34. 87830 „ 30389926 39. 6361504233 44. 940000
35. 340053 „ 1751 40. £1761 45. 7576 feet
36. 86985 „ 239999 41. 1412 46. 3883941
37. 40941296 „ 9507264342 42. 695 47. 39 48. 89

Second Course.—*Page* 13.

1. 2855 6. 979784 11. 1*d.* more. 16. 19999586937846
2. 65; 127 7. 57830 12. 162 17. 1999970999992
3. 12 *m.*; 40 *m.* 8. 867997 *g.* 13. 1759; 1911 18. 599999989450
4. 1044 9. 2000 14. C. 34; J. 22 19. 1268
5. £106 10. 79166 15. 495700 20. 2427902

Ex. 4.—*Page* 16.

1. 925468 12. 16526854 23. 3483342478
2. 1141782 13. 17831043 24. 6608198088
3. 1378148 14. 14346027 25. 4868330742
4. 1641789 15. 15614034 26. 1983568696
5. 2482473 16. 9862812 27. 3077232984
6. 3306196 17. 19154568 28. 7915165335
7. 3897012 18. 25891045 29. 129446; 109922; 165590
8. 3713375 19. 31712935 30. 147828; 252975; 207222
9. 4100320 20. 49203036 31. 328188; 254368; 321176
10. 3445578 21. 55328609 32. 48117490; 57740988; 410037815;
11. 12914582 22. 20821536 492045378
33. 49478382; 57724779; 2587540734; 3018797523
34. 4830329266; 5520376304; 578444706; 661079664
35. 12768905637; 17025207516; 21281509395; 25537811274; 29794113153
36. 338810366552; 592918141466; 677620733104; 762323324742
37. 5762115; 8052489; 8419158
38. 247217304; 494434608; 741651912; 12820548; 25641096; 38461644
39. 1685512386; 2528268579; 3371024772; 4213780965; 5056537158
40. 66317055763; 75790920872; 28421595327; 85264785981
41. 592932 42. 710682 43. 474488 44. 24768740 45. 10989000
46. 417216 47. 6847930; 7532723; 8217516; 821574960; 903732456; 985889952 48. 897300; 1096700; 1196400
49. 51760665560; 71170915145; 45290582365; 77640998340
50. 41965

Ex. 5.—*Page* 18.

1. 90622812; 103568928; 116515044; 142407276
2. 114990336; 134155392; 153320448; 158111712
3. 303115536; 168397520; 353634792; 252596280
4. 397169298; 463364181; 588398960; 411879272
5. 40278848; 45313704; 539284416; 606694968
6. *6390769*; 35427161; 59650591; 6927459
7. *4560605664*; 6270832788; 4180555192; 2327809141

8. 9229392; 8485488; 118994976 9. 272860176
10. 16519952 11. 17118076 12. 1839037712
13. 64938700; 649387000; 51950960; 389632200
14. 48876300; 217228000; 27153500; 3801490000
15. 6306166320 16. 3851237560 17. 534199680; 2713395200
18. 16757121600; 9774987600 19. 23781800; 332945200; 57076320
20. 6431910300; 7623004800; 786122870

Ex. 6.—*Page* 21.

1. 8252374 7. 28138196 13. 16652608 19. 74669220
2. 7298202 8. 24152971 14. 59162601 20. 33857703
3. 16623575 9. 27871697 15. 46291344 21. 41556357
4. 5890484 10. 47135751 16. 28079254 22. 37831905
5. 21545455 11. 41440250 17. 64270920 23. 62639444
6. 13212076 12. 21846265 18. 81901495 24. 63368466
25. 557265810; 797019705; 1587559575
26. 2957842602; 5549133408; 1296508712; 4779064864; 3787537660; 2509411968; 6408222645; 2243178354
27. 4376128800; 5467570056; 7224017688; 2314456203; 2733019028; 5570838462; 4746808442; 5747168700
28. 3000436912; 3481308708; 5309547423; 8641974250; 5677899784; 5788660212; 7011150972; 830690170
29. 223607853720; 390082070872; 409166758000; 619273481800
30. 228760086645; 514445478372; 352504534655; 432433595068
31. 23050369728; 70451512887 32. 61285065625; 34145734784
33. 50419372978; 97512336625 34. 44519857440; 76270551113
35. 43714481440; 87121640990 36. 31660459787024; 2162664515606 4; 45452451473088
37. 53498872850136; 46191494271010; 48665335214529
38. 36686637389250; 33969945399046; 21009206327826
39. 116728158855 40. 115782676016

Second Course.—Page 22.

1. 523 2. 439296 3. 2765760797712 4. 13646183040
5. 486 *s.* 6. 840 7. 2640 *d.* 8. 1080; 5832 9. 100026368
10. 901 *m.* 11. 634832 12. 103 *m.*; 37 *m.*; 29 *m.* 13. 509350 4
14. 7002641427

Ex. 7.—*Page* 24.

1. 231042 9. 175439 17. 85513 25. 54249 33. 115794
2. 263656 10. 275995 18. 78834 26. 131003 + 2 34. 49282
3. 314739 11. 38257 19. 165203 27. 125590 + 3 35. 34568
4. 471785 12. 85897 20. 157021 + 1 28. 103406 36. 72438
5. 421558 13. 77381 + 1 21. 205238 + 1 29. 121703 37. 43801
6. 387295 14. 136574 22. 125426 + 1 30. 73485 38. 62603
7. 481750 15. 236845 23. 69279 31. 39008 39. 91672
8. 157118 16. 197617 24. 123169 32. 56776 40. 81366 + 3
41. 58926 + 2 42. 275680 43. 42739 + 5 44. 67946
45. 46764 + 1 46. 28546 47. 114948 48. 32667

49. 17147814; 11431876; 8573907; 6859125+3; 5715938
50. 13656959; 10925567+1; 18209278+2; 7803976+4; 6828479+4
51. 13744653; 11781131+1; 20616979+2; 9163102; 10308489+6
52. 3704692; 2222815+1; 1234897+3; 1587725+1; 2778519
53. 6711701+3; 6040531+2; 7550664; 10067552; 5491392
54. 27586797; 11822913; 7523671+10; 9195599; 6896699+3
55. 17001182; 7556080+8; 5667060+8; 8500591; 6182248
56. 3077136; 5384988; 2692494; 3589992; 2393328
57. 3367170; 5387472; 4489560; 2693736; 2993040
58. 3891426; 3335508; 2594284; 2122596; 1945713
59. 1065886+1; 710590+5; 852709; 387595; 473727+2
60. 5452235+1; 4460919+7; 4089176+4; 6133764+4; 8178352+4
61. 262087+2; 157252+3; 112323+2; 87362+5
62. 4166816+7; 5555755+5; 6666907; 3703837+2; 3030412+3; 2777877+11 63. 694059+1
64. 4125177; 4714488 65. 36475824; 29843856; 54713736; 41035302; 65656483+1; 27356868
66. 2360776+2; 1888621+1; 1573851; 1049234; 858464+2; 1180388+2; 67. 221782; 110891; 177425+3; 126732+4; 98569+7 68. 20016720; 35029260; 15568560

Ex. 8.—*Page* 26.

1. 200368; 725096; 756349; 789423; 846309; 4525700
2. 570689; 841976; 847623; 8793046; 21428; 420916
3. 377899; 2970151; 976534; 394768; 546031; 128151+35
4. 402609; 887756; 319776; 657403; 390470; 146131
5. 246391; 894716; 987654; 162490; 190075; 15625
6. 382571; 149206; 847677; 360047; 676469; 185500+13
7. 526394; 427916; 84965; 425061; 842639; 114889+48
8. 325648+10; 500000 9. 425061 10. 289291+24
11. 659888127 12. 17891 13. 253923+21; 145099+15; 135426+3; 108824+29 14. 83823+41; 57046+56; 114093+20; 85570+8
15. 750222; 363744; 500148; 296384
16. 520074; 269668; 120347+85; 202251

Ex. 9.—*Page* 27.

1. 426351; 514172; 8345+9; 547136; 271469; 492617
2. 516073; 804176; 635104; 210053; 314278 3. 3343652; 1543224; 2507739; 3600856 4. 627336+8; 744961+32; 376401+65; 470502+8 5. 776970+44; 146308; 64100+66
6. 246496+14; 620809; 748005; 335069+2; 327640; 514630+96
7. 2116128; 1587096; 1166029+70; 1360368 8. 542763; 490050; 102490+72; 22759+175; 924006 9. 346791; 418060; 541673; 78572+601; 325064 10. 1543224; 3343652; *3600*856; 2507739 11. 805278+117 12. 576000 13. 723
14. *13161*+3728; 10131+4463; 7293+6080; 14374+796

15. 42567; 82471; 30450; 54639; 54197
16. 78848; 107520; 156053 + 432; 131413 + 513 17. 76547 + 595; 10400 + 2304; 675374; 13024 + 2432 18. 624372; 162219
19. 526578 rem. 10000; 9955452 rem. 10000 20. 162219

Second Course.—*Page* 28.

1. 2603527582 2. 13 3. 96 4. 14 *hrs.* 5. 153 6. 3 7. 80
8. £371 9. £9 10. Tom, 25; Harry, 11 11. 64800590 12. 70 *m.*
13. £1002. 14. A gives 17; C gives 5; B rec. 12; D rec. 10 15. 1
16. 1798; 1842; 1850; 1852 17. 17305204499807 61 rem. 1152
18. 2840 rem. 31014 19. div. 649513 rem. 683 20. 41

Ex. 10.—*Page* 32.

1. 47920194 2. 1422 3. 19008 4. A, 52; B, 26 5. £26.; £13.
6. 353485; 16293618 7. 49992 8. 1176 9. 1520
10. 6949696 11. 867000 12. A, 48*d.*; B, 24*d.*; C, 8*d.*
13. Sugar, 6*d.*; Coffee, 1*s.* 6*d.*; Tea, 4*s.* 6*d.* 14. 124; 124½

Ex. 11.—*Page* 36.

1. 6420 *s.*; 10080 *d.*; 16320 *f.* 2. 9596 *s.*; 227 *d.*; 708 *f.*
3. 6592 *d.*; 27537 *f.* 4. 15792; 118430; 27839 *hf. pence*
5. 949; 27210; 363331 *f.* 6. 17106; 9675 *sixpences.*
7. 5762; 38086 *fourpences.* 8. 163296 *hf. pence*; 6804 *fl.*
9. 7896 *hf. cr.*; 9870 *fl.*; 78960 *threepences.* 10. 5040 *gui.*; 10080 *hf. gui.*; 42336 *hf. cr.* 11. 136458 *sixpences*; 204687 *fourpences.* 12. £72.; 576 *hf. cr.*; 137 *hf. gui.*; and 18 *d. rem.* 13. £885. 1*s.* 6½*d.*; £133. 19*s.* 0½*d.* 14. 8960 *oz.*; 2720 *qrs.*; 25856 *drs.* 15. 1605184 *oz.*; 564992 *drs.* 16. 357754841 *drs.*
17. 21364 *tons*; 18 *cwt.* 18. 181 *tons* 7 *cwt.* 1 *qr.* 22 *lbs.* 10 *oz.* 13 *drs.*; £72. 6*s.* 2½*d.*
19. 846811000 *grs.*; 104 *hf. gui.* 20. 1825920 *grs.*; 4039000 *grs.*
21. 1431839 *grs.*; 97000 *mils.* 22. 459593 *grs.*; £32. 7 *fl.* 6 *c.* 9 *m.*
23. 605 *lbs.* 2 *oz.* 10 *dwt.*; 170 *lbs.* 3 *oz.* 16 *dwt.* 23 *grs.* 24. 2722 *lbs.* 2 *oz.* 5 *drs.* 1 *scr.*; 100000 *mils.* 25. 612 *in.*; 7360 *po.*; 10560 *yds.*
26. 252393 *in.* 27. 27777 *yds.* 2 *ft.* 4 *in.*; 136 *lbs.* 1 *oz.* 6 *dwt.* 16 *grs.*
28. 980 *na.*; 1584 *yds.* 29. 6272640 *sq. in.*; 42743 *sq. ft.* 36 *sq. in.*
30. 1205948 *sq. ft.*, 36 *sq. in.*; 10671 *scr.* 31. 14883264 *cub. in.*; 1071 *cub. yds.* 25 *cub. ft.* 531 *cub. in.* 32. 3492 *qts.*; 2016 *pks.*; 2313 *lbs.* 10 *oz.* 13 *dwt.* 8 *grs.* 33. 5502 *gals.*; 444 *na.*
34. 40320 *gills*; 100512 *pts.*; 22304 *qts.* 35. 117096 *hrs.*; 38880 *m.* 1814400 *sec.* 36. 10 *wks.* 15 *hrs.* 45 *m.* 35 *sec.*; 1476 *qts.*
37. 16830037 *sec.*; 3 *qrs.*, 2568 *grs. rem.* 38. 1875167651 *in.*; 20 *E. ells*, 2 *qrs.* 39. 16632 *pts.*; £3542. 9*s.* 4½*d.*; 212 *lbs.* 7 *oz.* 9 *dwt.* 4 *grs.* 40. 3176 *bar.* 10 *gals.*; 23607 *pun.* 46 *gals.* 1 *qt.* 1 *pt.* 2 *gills.*

Second Course.—Page 37.

1. 1431 *in.*; 228357½ *yds.* 2. 341 *fathoms*; 1743250 *mils.*; £1743. 5*s.* 3. 87840 *min.*; 7948800 *sec.*; 1464 *hrs.*
4. 250549056 *hf. pts.* 5. 6476; 30648. 6. 27439200 *sec.*
7. 98 *m.* 3 *fur.* 13 *po.* 4 *yds.* 2 *ft.* 8 *in.*; 4790 *ac.* 32 *po.* 8 *yds.* 116 *in.*
8. 3187249627 *sec.* 9. 19025 *yrs.* 319 *days* 10 *hrs.* 40 *min.*
10. £6383. 5*s.* 4*d.* 11. 271657767857 *tons* 2 *cwt.* 3 *qrs.* 12 *lbs.*
12. 2 *ro.* 37 *po.* 17 *yds.* 4 *ft.* 133 *in.*

Ex. 12.—*Page* 38.

1. £36. 8*s.* 3*d.* 2. £61. 13*s.* 1*d.* 3. £159. 4*s.* 11*d.* 4. £167. 10*s.* 6*d.*
5. £194. 11*s.* 7*d.* 6. £30. 18*s.* 5*d.* 7. £33. 5*s.* 11½*d.* 8. £83. 13*s.* 3*d.*
9. £146. 8*s.* 5½*d.* 10. £146. 11*s.* 7½*d.* 11. £179. 11*s.* 1*d.*
12. £183. 5*s.* 0½*d.* 13. 88 *lbs.* 2 *oz.* 6 *dwt.* 14. 55 *lbs.* 9 *oz.* 4 *drs.* 5 *grs.*
15. 53 *tons* 13 *cwt.* 2 *qrs.* 21 *lbs.* 16. £210. 19*s.* 1½*d.* 17. 58 *lbs.* 3 *oz.* 15 *dwt.* 12 *grs.* 18. 171 *ac.* 3 *ro.* 11 *po.* 19. £305. 15*s.* 7*d.*
20. 12 *m.* 5 *fur.* 27 *po.* 21. 93 *wks.* 1 *dy.* 10 *hrs.* 37 *m.* 26 *sec.*
22. £161. 4*s.* 6½*d.* 23. 51 *tons* 18 *cwt.* 2 *qrs.* 15 *lbs.* 4 *oz.* 24. 37 *qrs.* 5 *bus.* 2 *pks.* 1 *gal.* 1 *qt.*

Second Course.—Page 39.

1. £271. 10*s.* 1¾*d.* 2. £471. 2*s.* 6¾*d.* 3. £1316. 15*s* 7½*d.*
4. £761. 2*s.* 11*d.* 5. £11. 12*s.* 6½*d.* 6. £974. 19*s.* 3*d.*
7. 93 *ac.* 3 *ro.* 28 *po.* 8. £1279. 5*s.* 8¾*d.* 9. £11152. 13*s.* 10*d.*
10. £17. 12*s.* 6*d.* 11. £2086. 9*s.* 2*d.* 12. £2110. 5*s.* 3¼*d.*

Ex. 13.—*Page* 41.

1. £5. 1*s.* 5*d.*: £1. 9*s.* 1½*d.* 2. £16. 2*s.* 0½*d.*; £1. 7*s.* 10*d.*
3. £6. 17*s.* 2*d.*; £6. 0*s.* 11*d.* 4. £8. 5*s.* 0¾*d.*; £3. 18*s.* 8¾*d.*
5. £9. 15*s.* 11½*d.*; £56. 8*s.* 3*d.* 6. £55. 2*s.* 8*d.*; £30. 13*s.* 6*d.*; 18*s.* 11¼*d.*
7. £23. 19*s.* 9½*d.*; £16. 6*s.* 10¾*d.* 8. 3 *lbs.* 11 *oz.* 13 *dwts.* 10 *grs.*
9. 8 *wks.* 4 *d.* 20 *hrs.* 46 *m.* 9 *sec.* 10. £36. 15*s.* 10¾*d.*; 8 *qrs.* 1 *pk.*
11. £309. 15*s.* 5½*d.*; £276. 7*s.* 8½*d.*; £270. 17*s.* 0¾*d.*; £9. 3*s.* 10¾*d.*; £137. 18*s.* 11*d.*; £215. 19*s.* 6½*d.*; £97. 13*s.* 9½*d.*; £226. 14*s.* 6¾*d.*
12. £24. 16*s.* 11¾*d.*; 9 *lbs.* 6 *oz.* 6 *drs.* 11 *grs.*; 27 *ac.* 1 *ro.* 29 *po.*; £14. 16*s.* 2¾*d.*
13. £95. 13*s.* 6½*d.*; £419. 8*s.* 2¾*d.*; £443. 8*s.* 2¾*d.*; £5. 11*s.* 9½*d.*; £123. 3*s.* 7*d.*; £265. 7*s.* 5¾*d.* 14. £50.; £242. 10*s.*
15. £81666. 13*s.* 4*d.* 16. £1433. 9*s.* 8¾*d.*; £966. 13*s.* 3¾*d.*; £449. 15*s.* 7½*d.*; £258. 13*s.* 5½*d.*; £559. 19*s.* 11¾*d.*; £1368. 19*s.* 8½*d.*; £956. 10*s.* 6*d.*; £435. 9*s.* 3¾*d.*

Second Course.—Page 41.

1. £64. 13*s.* 2*d.* 2. £556. 17*s.* 9*d.* 3. £18. 5*s.* 11½*d.*
4. £879. 12*s.* 4*d.* 5. £331. 5*s.* 5*d.* 6. Equal. 7. 15720349 *miles* 497 *yds.* 2 *ft.* 4 *in.*
8. £46855. 0*s.* 10*d.* 9. 214 *lbs.* 15 *dwt.* 10. £276. 13*s.* 2½*d.*
11. 323. 14*s.* 11½*d.* 12. £83. 2*s.* 3½*d.*

Ex. 14.—*Page* 43.

1. 16*s.*; £1. 6*s.* 8*d.*; £1. 17*s.* 4*d.*; £2. 18*s.* 8*d.*; £2. 13*s.* 4*d.*
2. £2. 6*s.* 9*d.*; £3. 10*s.* 1½*d.*; £1. 11*s.* 2*d.*; £4. 13*s.* 6*d.*; £2. 14*s.* 6½*d.*
3. £10. 9*s.* 10*d.*; £26. 4*s.* 7*d.*; £47. 4*s.* 3*d.*; £36. 14*s.* 5*d.*; £57. 14*s.* 1*d.*
4. £69. 8*s.* 8*d.*; £104. 3*s.*; £26. 0*s.* 9*d.*; £60. 15*s.* 1*d.*; £86. 15*s.* 10*d.*
5. £38. 8*s.* 11½*d.*; £64. 1*s.* 6¾*d.*; £115. 6*s.* 9¾*d.*; £76 17*s.* 10½*d.*; £140. 19*s.* 5¼*d.* 6. £44. 15*s.* 5¾*d.* 7. £163. 7*s.* 0¾*d.*
8. £672. 4*s.* 4½*d.*; £1152. 7*s.* 6*d.* 9. £62. 0*s.* 2*d.*; £66. 8*s.* 9*d.*; £141. 14*s.* 8*d.*; £279. 0*s.* 9*d.* 10. £2002. 17*s.* 6*d.*; £1192. 3*s.* 9*d.*; £2575. 2*s.* 6*d.* 11. £1794. 15*s.* 9*d.*; £2791. 17*s.* 10*d.*; £3589. 11*s.* 6*d.* 12. 1 *ton* 10 *cwt.* 3 *qrs.* 4 *lbs.*; 3 *tons* 9 *cwt.* 1 *qr.* 2 *lbs.*; 5 *tons* 7 *cwt.* 3 *qrs.*
13. 47 *wks.* 19 *hrs.* 42 *m.*; 86 *wks.* 2 *da.* 16 *hrs.* 7 *m.*; 439 *wks.* 5 *da.* 7 *hrs.* 52 *m.* 14. £394. 0*s.* 7½*d.*
162 *lbs.* 4 *oz.* 4 *drs.* 1 *scr.* 7 *grs.* 15. £280. 9*s.* 1*d.*; £454. 10*s.* 7*d.*; £802. 13*s.* 7*d.* 16. £1221. 14*s.* 7½*d.*; £1114. 19*s.* 6½*d.*; £1055. 13*s.* 4¾*d.* 17. £899. 12*s.* 10½*d.*; £1220. 10*s.* 8½*d.*; £1954. 0*s.* 0½*d.* 18. 166 *lbs.* 7 *oz.* 4 *dwt.*; 315 *lbs.* 10 *oz.* 3 *dwt.*; 465 *lbs.* 1 *oz.* 2 *dwt.* 19. £1813. 19*s.* 9*d.*
20. 53 *tons* 1 *cwt.* 2 *qrs.* 7 *lbs.*

Second Course.—*Page* 43.

1. £4338. 8*s.* 2. 194877 *lbs.* 1 *oz.* 3. £45. 11*s.* 7½*d.*
4. £494. 11*s.* 9*d.* 5. £177. 9*s.* 6½*d.* 6. £17664. 17*s.* 0¾*d.*
7. £1247. 1*s.* 8*d.* 8. £508. 8*s.* 2*d.* 9. £19. 14*s.* 10. £18913. 15*s.* 6*d.*
11. £17216. 8*s.* 3*d.* 12. £10947. 9*s.* 0½*d.* 13. £116. 7*s.* 6*d.*
14. 1*s.* 0½*d.* 15. £5. 9*s.* 8*d.* 16. £10. 19*s.*

Ex. 15.—*Page* 46.

1. £17. 5*s.* 1*d.* 2. £14. 6*s.* 3*d.* 3. £10*s.* 6*s.* 7*d.* 4. £19. 8*s.* 2*d.*
5. £9. 14*s.* 1*d.* 6. £13. 9*s.* 11*d.* 7. £2. 3*s.* 11*d.* 8. £7. 17*s.* 2*d.*
9. 3 *cwt.* 3 *qrs.* 19 *lbs.* 10. 4 *lbs.* 11 *oz.* 16 *dwt.* 11. £7. 6*s.* 7*d.*
12. 4 *wks.* 1 *da.* 13. £6. 2*s.* 6*d.*; £9. 3*s.* 9*d.*; £5. 5*s.*; £3. 13*s.* 6*d.*
14. £3. 3*s.* 9*d.*; £8. 10*s.*; £5. 2*s.*; £2. 2*s.* 6*d.* 15. £9. 17*s.*; £5. 9*s.* 5½*d.*; £4. 2*s.* 1*d.*; £6. 3*s.* 1½*d.* 16. £72. 14*s.* 0¾*d.*; £56. 10*s.* 11½*d.*; £101. 15*s.* 8½*d.*; £46. 5*s.* 3¾*d.*
17. £19. 12*s.*; £17. 3*s.*; £7. 7*s.*; £6. 10*s.* 8*d.* 18. £33. 8*s.* 11¼*d.*; £15. 4*s.* 0¾*d.*; £27. 17*s.* 5½*d.*; £10. 2*s.* 8½*d.*
19. 15 *ac.* 3 *ro.* 18 *po.*; 6 *ac.* 27 *po.*; 7 *ac.* 8 *po.*; 8 *ac.* 36 *po.*
20. £10. 15*s.* 10*d.*; £30. 10*s.* 6*d.*; £19. 1*s.* 6¾*d.*; £12. 14*s.* 4½*d.*
21. £59. 13*s.* 5½*d.*; £72. 4*s.* 8½*d.*; £30. 9*s.* 11¾*d.*; £20. 9*s.* 8½*d.*
22. £114. 1*s.* 6*d.*; £214. 19*s.* 9*d.*; £71. 13*s.* 3*d.*; £103. 10*s.* 3*d.*
23. £1. 8*s.* 3½*d.*; 11*s.* 0¾*d.*; 8*s.* 7½*d.* 24. £2. 17*s.* 9¾*d.*; £15. 16*s.* 0½*d.*
25. £138. 8*s.*; £23. 1*s.* 4*d.*; £8. 13*s.* 26. 1 *qr.* 6 *lbs.* 2 *oz.*; 12 *lbs.* 13½ *oz.*; 9 *lbs.* 4$\frac{11}{12}$ *oz.*

27. 2970; 23. 28. 33; 192. 29. 400. 30. 35. 31. 97; 2160. 32. 365.

Second Course.—Page 47.

1. 467. 2. 129. 3. 49 *yds.* 4. £8316. 5. 10½*d.* 6. £115. 10*s.* 7. 15 *ft.*; 9 *ft.* 8. 117. 9. 39. 10. 1*d.* 11. 9½*d.* 12. 4*s.* 4½*d.*

Ex. 16.—*Page* 48.

1. £2. 10*s.* 11*d.* 2. £6. 0*s.* 10½*d.* 3. £2. 18*s.* 8*d.* 4. £2. 2*s.* 8½*d.* 5. £8. 16*s.* 5*d.* 6. £20. 9*s.* 6*d.* 7. £113. 12*s.* 6*d.* 8. £6. 16*s.* 2*d.* 9. £4. 0*s.* 7½*d.* 10. £18. 3*s.* 9½*d.* 11. £2. 17*s.* 12. £16. 3*s.* 13. £10. 4*s.* 8½*d.* 14. £2. 16*s.* 5½*d.* 15. £9. 16*s.* 16. £14. 3*s.* 1½*d.*

Ex. 17.—*Page* 52.

1. £6. 10*s.* 6*d.*; £9. 14*s.* 3*d.*; £22. 13*s.* 4*d.* 2. £15. 3*s.* 4*d.*; £5. 12*s.* 6*d.*; £7. 8*s.* 4*d.* 3. £5. 19*s.* 3*d.*; £18. 1*s.*; £3. 0*s.* 7½*d.* 4. £43. 6*s.* 8*d.*; £163. 6*s.* 8*d.*; £35. 5. £773. 13*s.* 10*d.*; £4528. 19*s.* 6*d.* 6. £438. 10*s.* 1*d.*; £660. 16*s.* 7*d.* 7. £5671; £2861. 17*s.* 6*d.* 8. £429. 6*s.* 8*d.*; £1374. 9*s.* 9. £375. 3*s.* 9½*d.*; £590. 15*s.* 9*d.* 10. £476. 10*s.* 3*d.*; £1523. 2*s.* 9½*d.* 11. £2341. 0*s.* 4*d.*; £2300. 15*s.* 4½*d.* 12. £1908. 18*s.* 11½*d.*; £808. 2*s.* 8*d.* 13. £137. 9*s.* 6*d.*; £11. 6*s.* 3*d.* 14. £144. 19*s.* 10½*d.*; £22. 0*s.* 4½*d.* 15. £9. 14*s.* 4½*d.*; £937. 5*s.* 10*d.* 16. £13. 18*s.* 1½*d.* 17. £178. 16*s.* 10½*d.* 18. £107. 2*s.* 4½*d.* 19. £1049. 19*s.* 8½*d.* 20. £6. 19*s.* 5*d.* 21. £2. 6*s.* 5½*d.* 22. £21. 6*s.* 3*d.* 23. £299. 11*s.* 11½*d.* 24. £61. 18. 2*d.*

Second Course.—Page 52.

1. £1018. 19*s.* 2*d.* 2. £104. 14*s.* 2½*d.* 3. £149306. 17*s.* 4. £2600. 3*s.* 5½*d.* 5. £16. 5*s.* 11½*d.* 6. £124. 8*s.* 0½*d.* 7. £157291. 13*s.* 4*d.* 8. £1092. 15*s.* 1*d.*

Ex. 18.—*Page* 55.

1. 16*s.* 8*d.* 2. 1*s.* 9*d.* 3. £7. 5*s.* 10*d.* 4. 7*d.* 5. £3. 8*s.* 9*d.* 6. 8 *days.* 7. 48 *men.* 8. 9 *lbs.* 9. 49 *yds.* 10. 54 *s.* 11. £46. 12. £288. 13. 30 *ac.* 14. £160. 17*s.* 6*d.* 15. 10 *days.* 16. 6 *days.* 17. £2000000. 18. 4 *cwt.* 3 *lbs.* 19. £10000. 20. 2 *lbs.* 21. £2000. 22. £300. 23. £2160. 24. 3 *months.*

Ex. 19.—*Page* 57.

1. £467. 2*s.* 3½*d.* 2. 89. 3. £2. 11*s.* 7½*d.*; £1. 5*s.* 9½*d.* 4. 4*s.* 6*d.* 5. 18. 6. 4*s.* 10½*d.* 7. 31 *wks.* 8. 2 *lbs.* 11 *oz.* 9. £764. 8*s.* 10. 4*s.* 8*d.* 11. £24. 6*s.* 12. £11. 13. £2000. 14. £126. 0*s.* 0½*d.* 15. £1500. 16. 2*d.* 17. 15*s.* 18. 5 *pints*; 3 *pints.* 19. 8 *oz.* 20. 1 *hr.* 53 *min.* 21. 600 *tons.* 22. £133. 6*s.* 8*d.* 23. 88 *lbs.* 24. 4*s.*

PART II.

Ex. 20.—*Page* 62.

1. 6 2. 13 3. 34 4. 19 5. 1 6. 13 7. 17
8. 97 9. 29 10. 53 11. 37 12. 97 13. 3 14. 11
15. 1 16. 59 17. 3 18. 67 19. 3 20. 30
21. 12 *feet*; 17 *feet*. 22. A's 5*s*.; B's 4*s*.

Ex. 21.—*Page* 64.

1. 72 2. 60 3. 72 4. 168
5. 462 6. 540 7. 288 8. 684
9. 8004 10. 78 11. 144 12. 5040
13. 1080 14. 2520 15. 18900 16. 3696
17. 180 18. 498960 19. 50388 20. 38640
21. 58198140 22. 146507309 23. £94. 10*s*. 24. 6 *wks*. 4 *dys*.
25. 36 *gals*. 26. 180 *miles*.

Ex. 22.—*Page* 67.

1. $\frac{0}{4}$; $\frac{5}{5}$; $\frac{21}{8}$; $\frac{8}{8}$; $\frac{9}{10}$.
2. $\frac{11}{2}$; $\frac{64}{17}$; $\frac{21}{2}$.
3. $\frac{1}{4}$; $\frac{1}{3}$; $\frac{2}{5}$; $\frac{2}{5}$; $\frac{5}{13}$; $\frac{3}{7}$.
4. $\frac{2}{10}$; $\frac{12}{65}$; $\frac{3}{40}$.
5. $\frac{0.3}{0}$; $\frac{7.2}{0}$; $\frac{0.1}{0}$; $\frac{0.0}{0}$; $\frac{1.52}{0}$.
6. $\frac{200}{33}$; $\frac{180}{13}$; $\frac{200}{13}$.
7. $\frac{144}{4}$; $\frac{840}{16}$; $\frac{978}{7}$.
8. $\frac{13}{32}$; $\frac{8}{13}$; $\frac{15}{44}$; $\frac{24}{35}$.

Ex. 23.—*Page* 68.

1. $3\frac{1}{4}$; $3\frac{3}{8}$; $3\frac{4}{5}$; $4\frac{5}{8}$; 5; $5\frac{3}{5}$; $8\frac{9}{11}$.
2. $\frac{23}{0}$; $\frac{30}{7}$; $\frac{35}{5}$; $\frac{0.0}{5}$.
3. $2\frac{5}{17}$; $5\frac{2}{13}$; $3\frac{3}{40}$; $3\frac{8}{103}$; $2\frac{170}{415}$.
4. $\frac{0.9}{5}$; $\frac{100}{7}$; $\frac{053}{11}$; $\frac{003}{5}$.
5. $6\frac{14}{15}$; $7\frac{1}{5}$; $3\frac{4}{5}$; $6\frac{2}{17}$; $3\frac{2}{7}$; $3\frac{10}{23}$; $5\frac{11}{12}$; $4\frac{13}{35}$.
6. $\frac{2}{5}$; $\frac{11}{13}$; $\frac{2}{5}$; $\frac{13}{17}$.
7. $1\frac{0}{35}$; $1\frac{2}{7}$; $1\frac{3}{11}$; $1\frac{31}{65}$; $1\frac{8}{19}$.
8. $\frac{1}{2}$.

Ex. 24.—*Page* 70.

1. $\frac{2}{5}$; $\frac{3}{5}$; $\frac{6}{11}$; $\frac{3}{5}$; $\frac{3}{5}$.
2. $\frac{4}{7}$; $\frac{7}{15}$; $\frac{42}{65}$; $\frac{144}{169}$.
3. $\frac{1}{2}$; $\frac{15}{16}$.
4. $\frac{1}{7}$.
5. $\frac{6}{11}$.
6. 16.
7. $\frac{11}{15}$.
8. 1.
9. $5\frac{1}{7}$.
10. 1.
11. $\frac{2}{3}$.
12. $248\frac{1}{5}$.
13. $\frac{11}{13}$; $\frac{23}{27}$; $\frac{3}{7}$; $\frac{10101}{143500}$.
14. $\frac{4}{5}$.
15. 3*d*.
16. 5.

Ex. 25.—*Page* 71.

1. $\frac{8}{12}$; $\frac{10}{12}$; $\frac{1}{12}$.
2. $\frac{27}{36}$; $\frac{12}{36}$; $\frac{8}{36}$.
3. $\frac{12}{42}$; $\frac{15}{42}$; $\frac{7}{42}$; $\frac{4}{42}$
4. $\frac{90, 105, 20, 18}{150}$.
5. $\frac{165, 48, 88, 198}{264}$.
6. $\frac{196, 54, 21, 96}{252}$.
7. $\frac{21, 75, 9, 4}{105}$.
8. $\frac{63, 30, 4, 15}{108}$.
9. $\frac{54, 70, 24, 55}{90}$.
10. $\frac{315, 836, 300, 35}{420}$.
11. $\frac{56, 81, 30, 26}{144}$.
12. $\frac{280, 693, 765, 546}{1890}$.

13. $\frac{128, 408, 594, 870, 465}{1440}$. 14. $\frac{1980, 612, 675, 280, 300, 522}{2340}$.

15. $\frac{27, 16, 30}{72}$. 16. $\frac{12}{45}; \frac{8}{45}$.

17. $\frac{1680, 3276, 4410, 7380, 7420, 1575}{21420}$.

18. $\frac{366795, 190190, 389025, 720720, 460845, 513513}{855855}$.

19. $\frac{11}{12}, \frac{37}{48}, \frac{17}{21}, \frac{3}{8}$ of $1\frac{1}{5}$. 20. $\frac{77}{85}, \frac{5}{8}$ of $1\frac{7}{17}, \frac{23}{48}$.

Ex. 26.—*Page* 72.

1. $\frac{7}{8}$	2. $\frac{11}{12}$	3. $\frac{2}{3}$	4. 1	5. $1\frac{1}{13}$
6. $2\frac{1}{4}$	7. $1\frac{17}{72}$	8. $5\frac{3}{4}$	9. $7\frac{1}{13}$	10. $2\frac{331}{1496}$
11. $6\frac{1}{6}$	12. $8\frac{2330}{3300}$	13. 12	14. $7\frac{31}{105}$	15. 4
16. $2\frac{23}{48}$	17. $9\frac{47}{130}$	18. 1	19. $\frac{2}{3}$	20. $7\frac{59}{315}$
21. $11\frac{1}{6}$	22. $14\frac{1}{4}$	23. $3\frac{233}{715}$	24. $4\frac{329}{300}$	25. $5\frac{91}{730}$
26. $49\frac{191}{400}$	27. $4\frac{8257}{10380}$	28. $12\frac{79}{144}$	29. $2\frac{10949}{80148}$	30. $1\frac{7733}{33510}$

Ex. 27.—*Page* 73.

1. $\frac{1}{6}$	2. $\frac{1}{6}$	3. $\frac{3}{20}$	4. $\frac{4}{13}$	5. $\frac{1}{36}$
6. $\frac{1}{8}$	7. $\frac{1}{336}$	8. $\frac{131}{1716}$	9. $\frac{8}{77}$	10. $\frac{7}{144}$
11. $\frac{7}{72}$	12. $\frac{3}{20}$	13. $2\frac{3}{5}$	14. $4\frac{11}{110}$	15. $2\frac{13}{72}$
16. $1\frac{35}{36}$	17. $\frac{115}{144}$	18. $1\frac{19}{36}$	19. $\frac{103}{364}$	20. $\frac{1}{30}$
21. $11\frac{7}{15}$	22. $1\frac{47}{72}$	23. $5\frac{356}{385}$	24. $25\frac{205}{378}$	25. $\frac{1}{2}$
26. $\frac{137}{210}$	27. $156\frac{1}{3}$	28. $\frac{3197}{4410}$	29. 2	30. $\frac{3}{5}$

31. $7\frac{243}{880}$ 32. $1\frac{3}{5}$

Second Course.—Page 74.

1. $\frac{7}{36}$	2. $9\frac{1770}{1320}$	3. $1\frac{593}{1300}$	4. 186
5. 6 *gals.*	6. $\frac{227}{405}$	7. $\frac{23}{240}$	8. 241 *times.*
9. $\frac{9}{40}$	10. $\frac{1}{15}$	11. $4\frac{1}{5}$ of $3\frac{6}{7}$ of $\frac{5}{36}$	12. 2 *gals.*

Ex. 28.—*Page* 75.

1. $\frac{3}{8}$	2. $\frac{6}{35}$	3. $\frac{24}{77}$	4. $\frac{9}{20}$	5. $\frac{1}{3}$
6. $\frac{1}{2}$	7. $\frac{5}{6}$	8. $\frac{1}{13}$	9. $\frac{2}{63}$	10. $\frac{25}{156}$
11. $\frac{3}{8}$	12. $1\frac{3}{7}$	13. $\frac{9}{40}$	14. $\frac{1}{13}$	15. $1\frac{1}{7}$
16. 1	17. $1\frac{3}{5}$	18. $5\frac{1}{2}$	19. $\frac{9}{35}$	20. 2
21. $195\frac{1}{15}$	22. 98	23. $\frac{76}{105}$	24. $\frac{1024}{1035}$	25. $19\frac{1}{5}$
26. $1\frac{43}{52}$	27. 8	28. 1	29. $1\frac{9}{56}$	30. 2

31. $\frac{3991}{4752}$ 32. $19\frac{1019}{20160}$

Ex. 29.—*Page* 77.

1. $\frac{3}{5}$	2. $1\frac{1}{3}$	3. $\frac{1}{3}$	4. $1\frac{1}{8}$	5. 4
6. $1\frac{1}{5}$	7. 6	8. 6	9. $1\frac{1}{8}$	10. $3\frac{1}{3}$

11. $\frac{68}{361}$ 12. $\frac{333}{400}$ 13. $1\frac{117}{125}$ 14. $6\frac{8}{7}$ 15. $\frac{4}{65}$
16. 1 17. $2\frac{113}{225}$ 18. 2 19. $15\frac{1}{6}$ 20. $4\frac{459}{664}$
21. $\frac{1}{14}$ 22. $\frac{100}{4003}$ 23. $1\frac{1}{325}$ 24. $43\frac{1}{2}$ 25. $3\frac{2}{5}$
26. $\frac{3}{7}$ 27. 8 28. 3 29. $2\frac{247}{250}$ 30. $2\frac{3404}{4445}$
31. $18\frac{3}{7}$ 32. $5\frac{111}{1049}$

Ex. 80.—*Page* 78.

1. $\frac{3}{35}$; $11\frac{1}{2}$; $\frac{13}{48}$; $\frac{4}{7}$; $\frac{5}{6}$; $\frac{1}{12}$; $1\frac{4}{21}$; 3; $\frac{3}{5}$; $1\frac{3}{5}$.
2. $\frac{3}{5}$; 3; 12; $\frac{4}{5}$; $6\frac{6}{7}$; 9; $\frac{1}{12}$. 3. $2\frac{1}{16}$; 1; $2\frac{3}{4}$; $\frac{1225}{6144}$.
4. 5; $\frac{4}{37}$; $\frac{42}{3411}$; $1\frac{157}{2356}$. 5. $3\frac{24}{56}$; $1\frac{35}{44}$; $\frac{65}{67}$. 6. $5\frac{1}{5}$; $1\frac{128}{175}$; $\frac{1}{37}$.

Second Course.—*Page* 79.

1. $\frac{1}{10}$. 2. $\frac{3}{5}$. 3. $7\frac{511}{880}$. 4. £5. 5. $2\frac{1237}{8000}$.
6. The former is already the greater fraction by $\frac{65}{1408}$.
7. $\frac{3}{5}$; *or* 3 *quarts*. 8. 3 *yds*. 1 *ft*. $3\frac{3}{4}$ *in*. 9. $5\frac{1}{2}\frac{4}{5}\frac{2}{3}\frac{9}{0}$.
10. $11102\frac{403}{544}$. 11. $22\frac{257}{405}$. 12. $7\frac{33}{64}$ and $3\frac{11}{13}$. 13. $\frac{13}{30}$. 14. $4\frac{44}{69}$.
15. $\frac{25}{64}$. 16. $8\frac{1}{3}$ *gals*.

Ex. 31.—*Page* 82.

1. $\frac{8}{10}$; $\frac{71}{100}$; $\frac{9}{100}$; $\frac{307}{1000}$; $\frac{19}{1000000}$; $5\frac{11}{100}$; $29\frac{7}{10}$; $\frac{1}{10000}$; $314\frac{13}{10000}$; $\frac{3000001}{10000000}$. 2. ·21; ·0019; 59·1; 39·03. 3. $\frac{1}{5}$; $\frac{3}{20}$; $\frac{4}{5}$; $\frac{13}{25}$; $4\frac{1}{2}$; $31\frac{1}{400}$; $\frac{7}{80}$. 4. 3974·1; 39741; 39741000.
5. 4·113; ·04113; ·00004113. 6. 5170030. 7. $3\frac{1}{5}$; $31\frac{1}{8}$; $312\frac{1}{3}$.
8. $241\frac{7}{8}$; $2\frac{67}{160}$; $\frac{887}{10000}$.

Ex. 32.—*Page* 83.

1. 150·328 2. 845·2864 3. 630·39587 4. 3686·3734
5. 3995·054 6. 2 7. 1210·03941 8. 6913·42378
9. 530 10. 2·959 11. 1·9 12. 6090·444843007

Ex. 33.—*Page* 84.

47·05 2. 619·641 3. ·1686 4. ·001866
5. 5·0124 6. ·667 7. 3116·033 8. 1·10909
9. 8599·00007 10. 3411·3265108 11. 2·941 12. 2·10071

Ex. 34.—*Page* 85.

1. 258·453 2. 81·64 3. 2·34872 4. ·259
5. 1·8981 6. ·003615 7. 473·701146 8. ·0000712
9. 3080·892 10. 5690·31 11. ·0756 12. 16·0006
13. 3·26716 14. ·0396 15. ·0000004 16. 5062·140422
17. 80641 18. ·07056063 19. ·2548 20. 27895
21. 13·49513 22. 1587·01283 23. 6·573 24. 6·573
25. 133·238; 13·3238; 1332·38. 26. ·06156; 6·156; 6·156.
27. 224; 2·24; 2240. 28. ·00319; 31·9; 31900; ·000319; ·000319
29. ·0001. 30. 8·1. 31. 4862550·756. 32. $5148\frac{21}{40}$.

Ex. 35.—*Page* 87.

1. 2·4 2. 5·2 3. ·17 4. 4·8 5. 3·16
6. ·03 7. 2·82 8. 3·174 9. ·00763 10. 3·1
11. 6·3 12. ·004 13. ·22 14. ·33 15. ·0019375
16. ·072 17. ·015625 18. 23 19. 5·3 20. ·0009
21. 8 22. 2·46875 23. ·0003125 24. ·0016 25. 32·58
26. 7·19 27. 713·4 28. ·644 29. 72·599609375
30. ·38072 31. 67000 32. 210 33. 20937·5 34. 51600
35. 31000 36. 427500 37. 65·526 38. 5·2
39. 770; ·28; 3 40. 200200; 23·78613, &c.; 1001.000
41. 39200; 29·4 42. 17; ·017; ·017; 17000; 170000000; 170
43. 86·25; 8625; ·000008625; 86250000; 8625
44. 387·6953125; ·00003876953125; ·3876953125; 3·876953125; 38769531·25; 45. ·7468933; 110·9185441; 4385·9649122
46. ·0014542; ·0024848; 1329·0789473
47. 1790368·4210526; ·0322580; ·0001567
48. 3·1415929; 273076·9230769; ·0003141

Second Course.—*Page* 88.

1. 1·3 2. ·000027 3. 135252 4. ·5777994, &c.
5. 1260 6. ·03 7. ·143 8. 12
9. 35·1 10. 719000 11. ·0194 12 10·53634725

Ex. 36.—*Page* 89.

1. ·5; ·75; ·625; 3·5; 7·875; ·6875; ·46875; ·725; 5·2375.
2. ·984375; ·6171875; 29·52; ·368; ·923828125; 259·50625.
3. ·3125; 1·2152777, &c.; 1·86875; 21·75; 4. 7·421875; 25·1484375.
5. 2·675. 6. 3·1125. 7. 7·95625. 8. 3·67421875. 9. 2·1375.
10. 1·6640625. 11. ·910859375. 12. 7·436. 13. ·0908203125.
14. 19·514625. 15. ·7421875. 16. ·18. 17. 1·2. 18. 20.
19. ·028125. 20. ·0076. 21. 1·059. 22. 1·09375.

Ex. 37.—*Page* 91.

1. $\dot{6}$; $\dot{8}$; $\cdot\dot{1}\dot{6}$; $\cdot\dot{6}\dot{3}$; $\cdot4\dot{1}\dot{6}$; $\cdot\dot{4}\dot{6}$; $\cdot\dot{7}\dot{2}$; $\cdot\dot{7}0\dot{8}\dot{3}$; $\cdot\dot{3}\dot{6}$. 2. $2\cdot\dot{1}$; $3\cdot\dot{8}\dot{3}$; $\cdot857142\dot{8}$; $29\cdot8518\dot{3}$; $\cdot538461\dot{5}$; $\cdot\dot{5}4\dot{6}$; $\cdot8628257$, &c.; $4\cdot99\dot{3}$.
3. $\cdot364583\dot{3}$; $\cdot714285\dot{7}$; $137\cdot285714\dot{3}$;
4. $\frac{2}{9}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{21}{99}$; $\frac{118}{999}$; $\frac{4233}{9999}$; $53\frac{205}{999}$.
5. $\frac{3}{25}$; $\frac{21}{50}$; $\frac{7}{500}$; $\frac{2}{5}$; $\frac{42}{75}$; $8\frac{1}{20}$; $71\frac{7}{15}$. 6. $\frac{33}{88}$; $\frac{173}{990}$; $6\frac{511}{1000}$; $78\frac{13}{20}$; $\frac{21}{880}$. 7. $\frac{170}{999}$; $\frac{8547}{11111}$; $\frac{4193}{13310}$; $46\frac{13}{88}$; 10.
8. $\frac{8744317}{10000000}$; $\frac{791}{800000}$; $\frac{114027}{800000}$.

Ex. 38.—*Page* 93

1. 81·61737 2. 43·46881 3. 206·03346 4. 40·20447
5. *3·79535* 6. ·01865 7. 3·93222 8. ·00333

9. 14·20720 10. ·26623 11. ·37087 12. ·09523
13. 1·45833 14. 1·94635 15. 1·25 16. $1{\cdot}0\dot{6}$
17. ·47619 18. ·375 19. 1 20. ·5

Second Course.—*Page* 93.

1. $\cdot45\dot{8}\dot{3}$ 2. $34{\cdot}2\dot{8}5714\dot{2}$ 3. ·5 4. $\cdot0\dot{6}$, or $\frac{1}{15}$ 5. 2·4
6. $\cdot2\dot{7}$ 7. 1 8. 36·4 9. 120 10. 3600
11. 6·307, &c. 12. 1279500 13. ·718276 14. 15925 15. 72
16. 36 *miles.* 17. 6·9504, &c. 18. 3·19959, &c. 19. 10 *min.* 40 *sec.*
20. $\cdot840\dot{9}$ *oz.*

Ex. 39.—*Page* 95.

1. £45. 6*fl.* 8*c.* 3*m.* 2. £225. 9*fl.* 3*c.* 7*m.* 3. £262. 6*fl.* 8*c.*
4. £15. 7*fl.* 4*c.* 6*m.* 5. £166. 9*fl.* 8*c.* 9*m.* 6. £2. 4*fl.* 1*c.* 3*m.*
7. £7. 7*c.* 7*m.* 8. £140. 1*c.* 2*m.* 9. 1*m.*
10. £22. 5*fl.* 5*m.* 11. £158. 2*fl.* 6*c.* 2*m.* 12. £29. 8*fl.*
13. £679. 5*fl.* 14. £1008. 6*c.* 15. £2700.
16. £1. 3*fl.* 7*m.* 17. 3*fl.* 7*c.* $8\frac{1}{2}$*m.* 18. 1*c.* 6*m.*
19. £31. 4*fl.* 2*c.* 5*m.* 20. £44. 2*fl.* 8*c.*

Ex. 40.—*Page* 96.

1. 3*s.*; 1*s.* 2*d.*; 6*s.* 3*d.*; 1*s.* 2*d.* 2. 6*s.* 4*d.*; £1. 2*s.*; £3. 9*s.*; 1*s.* $4\frac{1}{2}$*d.* 3. 1*s.* 7*d.*; £2. 12*s.* 6*d.*; 7*s.* 3*d.*; £1. 0*s.* 5*d.*
4. 5*s.*; 9*s.* 3*d*; £1. 9*s.* $1\frac{1}{2}$*d.* 5. 1120 *yds.*; 2 *qrs.* 24 *lbs.*; 5*s.* $2\frac{1}{2}$*d.*; £1. 9*s.* 7*d.* 6. 2 *qrs.* 6 *lbs.* 8 *oz.*; 40 *gals.* 2 *qts.*
7. 10 *dys.* 16 *hrs.* 40 *min.*; £5. 12*s.* 6*d.* 8. £6. 18*s.* 8*d.*
9. 11*s.* $7\frac{21}{44}$*d.* 10. 32 *ac.* 2 *ro.* 3 *po.* $8\frac{1}{4}$ *yds.* 11. 1 *lb.* 11 *oz.* 14 *grs.*
12. 90 *gals.*

Ex. 41.—*Page* 97.

1. $\frac{1}{2}$; $\frac{1}{6}$. 2. $\frac{1}{6}$; $\frac{3}{6}$; $\frac{2}{3}$; $\frac{5}{6}$. 3. $\frac{1}{3}$; $\frac{1}{8}$. 4. $1\frac{13}{24}$; $\frac{7}{16}$.
5. $\frac{1}{6}$; $\frac{7}{18}$; $\frac{5}{18}$. 6. $\frac{1}{15}$; $\frac{7}{15}$; $\frac{1}{5}$. 7. $\frac{2}{7}$; $\frac{9}{10}$. 8. $\frac{1}{4}$.
9. $\frac{3}{10}$. 10. $\frac{29}{45}$. 11. $\frac{3}{35}$. 12. $\frac{25}{45}$.

Ex. 42.—*Page* 99.

1. 5*s.*; 9*d.*; 7*s.* 6*d.*; 7*s.* 6*d.* 2. 4*d.*; £8. 16*s.*; 2*d.*
3. £4. 6*s.*; 2*s.* $7\frac{1}{2}$*d.*; 9*s.* $4\frac{1}{2}$*d.* 4. 7 *cwt.* 7 *lbs.*; 16 *cwt.* 1 *qr.* 11·2 *lbs.*; 1 *dwt.* 19·2 *grs.* 5. 16 *ac.* 32 *po.*; 50 *min.* 24 *sec.*; 5 *fur.* 180 *yds.*
6. £1. 2*s.* 2*d.* 7. £2. 16*s.* 9*d.* 8. 2 *qrs.* 3 *na.* $\frac{1}{4}$ *in.*; $1\frac{1}{2}$*d.*; 14*s.*
9. £16. 5*s.* $8\frac{1}{4}$*d.*; £2. 9*s.* $4\frac{1}{2}$*d.* 10. 3*s.* 6*d.* 11. £10. 1*s.*
12. £1401. 13*s.* 4*d.*

Ex. 43.—*Page* 100.

1. ·35; ·25. 2. ·2; ·625. 3. ·625; 1·5625. 4. ·125; ·375.
5. 1·09714285; ·4625. 6. ·5; ·10416. 7. ·1. 8. ·0452.
9. ·175. 10. ·546875. 11. 1·33904761. 12. 13·63.

Second Course.—*Page* 101.

1. ·25. 2. 8·75. 3. £126. 4. £3. 6*s*. 5. ·0416. 6. $\frac{4}{7}$.
7. A, 7*s*. 6*d*.; B, 12*s*. 8. 1*d*. 9. 4*s*. 2*d*. 10. ·32.
11. £448. 14*s*. $2\frac{2}{3}\frac{2}{3}d$. 12. 13*s*. $8\frac{1}{2}d$. 13. 5 *cwt*. 3 *qrs*. 22 *lbs*.
14. 14. 15. 88 *yds*. 16. 85 *fr*. 85 *c*.

Ex. 44.—*Page* 105.

1. 140 *sq. in*. 2. 231 *sq. ft*. 3. 5*s*. 10*d*. 4. £1270. 10*s*.
5. £6. 6. £3. 7*s*. $8\frac{1}{2}d$. 7. 76 *yds*. 8. 21 *ft*.
9. 45 *cub. ft*. 10. £21. 15*s*. 11. £4. 16*s*. 3*d*. 12. $73\frac{1}{2}$ *sq. ft*.
13. £3. 14. 116 *yds*. 15. £11. 15*s*. 16. 16 *ac*. 1 *ro*.
7 *po*. $28\frac{1}{4}$ *yds*. 17. 288. 18. £1. 2*s*. $5\frac{1}{2}d$. 19. 88 *cub. yds*. 14 *ft*.
20. 2 *cwt*. 1 *qr*. 21 *lbs*. 21. $970\frac{1}{2}$ *sq. ft*. 22. £7. 4*s*. 23. £96.
24. 1980 *sq. yds*. 25. $141\frac{1}{2}$ *sq. in*. 26. £8. 17*s*. 1*d*. 27. £3. 9*s*. 4*d*.
28. $16\frac{1}{2}$ *sq. ft*. 29. 24 *sq. in*. 30. 33 *cwt*. 3 *qrs*. 31. £21. 1*s*. 8*d*.
32. 468 *cub. yds*. 14 *ft*.

Ex. 45.—*Page* 107.

1. 9 *sq. ft*. 48 *in*. 2. 18 *sq. ft*. 112 *in*. 3. 28 *sq. ft*. 112 *in*.
4. 153 *sq. ft*. 108 *in*. 5. 165 *sq. ft*. 54 *in*. 6. 104 *sq. ft*. 26 *in*.
7. 32 *sq. yds*. 7 *ft*. 45 *in*. 8. 14 *sq. ft*. 9. 17 *cub. ft*. 415 *in*.
10. £1. 0*s*. $0\frac{1}{4}d$. 11. 4*s*. 8*d*. 12. £2. 16*s*. 3*d*.

Ex. 46.—*Page* 109.

1. 1, 4, 4, 16. 2. 2, $4\frac{1}{2}$, $4\frac{1}{2}$, 18. 3. $\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$, 6.
4. $1\frac{1}{7}$, $5\frac{1}{4}$, $5\frac{1}{4}$, $9\frac{1}{7}$. 5. $\frac{1}{2}$, 48, 48, 3. 6. $2\frac{2}{3}$, 24, 24, $13\frac{1}{3}$.
7. $10\frac{1}{3}$, $13\frac{1}{3}$, $13\frac{1}{3}$, 24. 8. $14\frac{1}{6}$, $15\frac{1}{6}$, $15\frac{1}{6}$, 40. 9. $5\frac{1}{11}$, $8\frac{3}{11}$, $8\frac{3}{11}$, $20\frac{1}{11}$.
10. $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{5}$, $\frac{1}{6}$. 11. $\frac{2}{5}\frac{5}{6}$, $9\frac{3}{4}$, $9\frac{3}{4}$, $1\frac{1}{1}\frac{1}{3}$. 12. 1; ·0004; ·0004; ·16.

Ex. 47.—*Page* 112.

1. £2. 14*s*. 2. 88 *ac*. 3. 9 *yds*. 4. 6 *dys*. 5. 12 *men*.
6. 6*d*. 7. 15 *lbs*. 8. 5 *lbs*. 9. 11*s*. $4\frac{1}{2}d$. 10. 432 *ac*.
11. 18*s*. 12. 2 *tons* 17 *cwt*. 13. £2240. 14. £315. 17*s*. 6*d*.
15. 16*s*. 3*d*. 16. £233. 15*s*. 17. £13. 10*s*. 18. 3 *tons* 18 *cwt*.
19. £720. 20. £146. 12*s*. 6*d*. 21. £4. 7*s*. 6*d*. 22. £27. 2*s*. $11\frac{1}{4}d$.
23. £2070. 24. $10\frac{1}{3}d$. 25. 26 *yds*. 26. 15 *dys*. 27. £5. 8*s*.
28. $59\frac{3}{5}$. 29. 6*d*. 30. £15444. 10*s*. 31. $4\frac{1}{2}$. 32. £2000.
33. £3. 4*s*. 6*d*. 34. $22\frac{1}{2}$ *dys*. 35. £31. 13*s*. $11\frac{1}{4}d$. 36. 6 *cwt*. 1 *lb*.

37. £798. 5*s.* 10*d.* 38. 1*s.* 3*d.* 39. 23*s.* 40. £227. 10*s.* 41. 33$\frac{1}{3}$%
42. 4*s.* 6*d.* 43. £430. 44. ·0391. 45. 4; 9; 16. 46. 120.
47. £1. 4*s.* 0$\frac{3}{4}$*d.* 48. £4. 10*s.*

Ex. 48.—*Page* 118.

1. 63$\frac{3}{4}$ *miles.* 2. 4 *horses.* 3. 36 *miles.* 4. 12 *men.*
5. £8. 6*s.* 8*d.* 6. 41$\frac{1}{4}$ *miles.* 7. 104. 8. 4 *cwt.* 1 *qr.* 18 *lbs.*
9. 42*s.* 10. 4 *mo.* 11. 14 *wks.* 12. 240000.
13. 10 *dys.* 14. 5 *men.* 15. 66 *lbs.* 16. £169. 6*s.* 3*d.*
17. 3 : 2 that is, one-half. 18. £1. 0*s.* 7$\frac{1}{2}$*d.* 19. 698 *miles.*
20. 3 *hrs.* 21. diminished one-third, or 2 : 3. 22. 9 *hrs.* 23. 6 *lbs.*
24. 35 *dys.* 25. 10 *mo.* 26. £8. 8*s.* 5$\frac{1}{2}$*d.* 27. 12 *ac.* 28. £442.
29. 27$\frac{1}{2}$ *miles.* 30. £5. 13*s.* 9*d.* 31. 272 *men.* 32. 108*s.* 33. £1. 14*s.* 8$\frac{1}{2}$*d.*

Ex. 49.—*Page* 123.

1. £67. 2. £174. 8*s.* £81. 4. £103. 10*s.*
5. £49. 9*s.* 7*d.* 6. £4. 1*s.* 10$\frac{1}{2}$*d.* 7. £946. 8*s.* 8. £2259. 8*s.*
9. £462. 3*s.* 6*d.* 10. £385. 11. £66. 13*s.* 4*d.* 12. £26. 13*s.* 4*d.*
13. 20 *yrs.* 14. 3$\frac{1}{2}$ *yrs.* 15. 2$\frac{1}{2}$ *yrs.* 16. 3$\frac{1}{2}$ *p. c.* 17. 4 *p. c.*
18. 4 *p. c.* 19. £125. 20. £300. 21. £368. 6*s.* 8*d.*
22. 5 *yrs.* 23. 11$\frac{1}{2}$ *mo.* 24. 1$\frac{1}{4}$ *yrs.* 25. 2$\frac{3}{8}$ *p. c.* 26. 10 *p. c.*
27. 6 *p. c.* 28 20 *yrs.* 29. 25 *yrs.* 30. 2 *p. c.*; 6$\frac{1}{4}$ *p. c.*
31. £522. 8*s.* $1\frac{3}{20}\frac{7}{10}\frac{1}{0}$*d.* 32. 2$\frac{1}{2}$ *p. c.*

Ex. 50.—*Page* 125.

1. £30. 15*s.* 2. £123. 0*s.* 6*d.* 3. £993. 16*s.* 9*d.* 4. £431. 0*s.* 3*d.*
5. £74. 5*s.* 6. £3. 12*s.* 7. £141. 7*s.* 6*d.* 8. £1750

Ex. 51.—*Page* 127.

1. £20. 5*s.* 2. £24. 3. 17*s.* 3*d.*
4. £1. 0*s.* $5\frac{7}{10}$*d.* 5. 7*s.* 5$\frac{1}{2}$*d.* 6. £21. 15*s.* $1\frac{9}{10}$*d.*
7. £215. 12*s.* 8. £717. 16*s.* 9. £418. 6*s.* 3*d.*
10. £254. 1*s.* 6$\frac{1}{4}$*d.* 11. £222. 13*s.* 9$\frac{3}{4}$*d.* 12. £141. 3*s.* 1$\frac{1}{5}$*d.*
13. £4. 6*s.* 8*d.* 14. £4. 8*s.* 15. £4. 1*s.*
16. £16. 17*s.* $7\frac{4}{8}\frac{3}{17}$*d.* 17. £1. 14*s.* 2*d.* 18. £7. 8*s.* $2\frac{4}{3}\frac{8}{0}\frac{4}{7}\frac{4}{0}\frac{3}{3}$*d.*
19. £640. 20. £666. 13*s.* 4*d.* 21. £120.
22. £30. 23. £1481. 14*s.* $7\frac{3}{7}\frac{1}{1}\frac{1}{3}$*d.* 24. £205. 6*s.* 3*d.*
25. £3. 15*s.* 26. £126. 14*s.* $6\frac{3}{10}$*d.* 27. £3. 11*s.* 5$\frac{1}{2}$*d.*
28. £204. 1*s.* 6*d.* 29. £371. 7*s.* $7\frac{3}{11}\frac{4}{11}$*d.* 30. £1. 0*s.* $9\frac{9}{14}$*d.*
31. £2. 6*s.* $1\frac{2}{2}\frac{8}{0}\frac{1}{1}\frac{3}{0}\frac{8}{5}\frac{4}{7}$*d.* 32. £5. 17*s.* 3·12*d.*; or, 4·53 *p. c.*

Ex. 52.—*Page* 131.

1. £3000. 2. £1400. 3. £1200. 4. £1200. 5. £3000.
6. £599. 4*s.* 8*d. nearly.* 7. 92. 8. 89$\frac{1}{4}$. 9. 93$\frac{4}{5}$.
10. £501. 17*s.* 6*d.* 11. £3233. 19*s.* 10$\frac{1}{2}$*d.* 12. £4859. 5*s.* 10$\frac{1}{2}$*d.*
13. £637. 4*s.* 14. £1776. 11*s.* 6*d.* 15. £17. 16. £95.

17. £142. 19*s.* $5\frac{19}{197}$*d.* 18. £142. 16*s.* 19. £168. 20. £3100.
21. £23417. 2*s.* 22. £5775. 23. £28875. 24. £31891. 5*s.* 25. $3\frac{1}{3}$.
26. $3\frac{4}{7}$. 27. $6\frac{1}{2}$. 28. £2685. 29. £3514. 10*s.* 30. £318. 15*s.*
31. increased £8. 1*s.* $10\frac{1}{4}$*d.* 32. £36.

Ex. 53.—*Page* 133.

1. £8. 1*s.* 2. 20 *p.c.* 3. £74. 2*s.* $4\frac{4}{17}$*d.* 4. $12\frac{24}{41}$. 5. 27*s.* 6*d.*
6. 7*s.* $10\frac{1}{2}$*d.* 7. 8 *p.c.* 8. 15*s.* $8\frac{4}{25}$*d.* 9. £60. 10. 2*s.* $7\frac{1}{4}$*d.*
11. $93\frac{1}{10}$; £249. 7*s.* 6*d.* 12. 10*s.* 9*d.* 13. $38\frac{5}{9}$. 14. $6\frac{68}{257}$.
15. £39. 1*s.* 3*d.* 16. 18 *p.c.* 17. $37\frac{1}{2}$ *p.c.* 18. 4*d.* 19. £285 15*s.*
20. $9\frac{102}{116}$ *p.c.*; £145. 21. £20. 22. £1. 16*s.*; $2\frac{2}{7}$ *p.c.*
23. £575; 40 *p.c.* 24. £25. 18*s.*; $4\frac{1}{2}$ *p.c.*

Ex. 54.—*Page* 137

1. 6318 *fl.* 2. 2097 *thal.* 25*s. gro.* 3. 1714 *fr.* 50 *c.*
4. 2927 *R. dal.* 24 *sk.* 5. 5192 *roub.* 46 *cop.* 6. 10055 *rup.* 7·8 *an.*
7. 8647 *l. nuo.* 54 *c.* 8. 1140 *milr.* 997 *r.* 9. 1257 *mks.* 3 *sch.*
10. 833 *h. dol.* 8 *ris.* 11. 1067 *fl.* 1 *kr.* 12. 47725 *pias.* 26 *par.*
13. £840. 14. £125. 15. £63. 10*s.* 16. £1. 1*s.* 8*d.* 17. £33. 1*s.*
18. £53. 19*s.* $5\frac{1}{2}$*d.* 19. £13. 1*s.* 3*d.* 20. £73. 2*s.* $2\frac{1}{2}$*d.* 21. £97.
22. £57. 23. £105. 15*s.* 24. £71. 12*s.* 25. 1496 *fl.* $8\frac{1}{2}$ *stv.*
26. £82. $5\frac{5}{159}$*s.* 27. 52 *l. nuo.* $70\frac{5}{8}$ *c.* 28. 140 *roub.*
29. $12874\frac{3}{127}$ *pias.* 30. 5 *fr.* 71 *c.* 31. 371 *fr.* 70 *c.* 32. $100\frac{100}{185}$ *fr.*
33. $34\frac{2}{3}$ *sch.* 34. 3772 *fr.* 83 *c.* 35. 100 *fl.* 49 *kr. for* 100 *fl.*
36. *Through Amsterdam.* 37. *Direct.* 38. 4 *fr.* 80 *c.* 39. 3*s.* $7\frac{7}{11}$*d.*
40. 121·11 *p.c.* 41. 5*s.* 11*d. nearly.* 42. £1. 0*s.* 5*d.* 43. 13*s.* 8·67*d.*
44. £461. 7*s.* 6*d.*

Ex. 55.—*Page* 140.

1. 33; 99. 2. A has £6. 4*s.* $3\frac{3}{4}$*d.*; B, £24. 17*s.* 3*d.*; C, £12. 8*s.* $7\frac{1}{2}$*d.*
3. A's, £113. 1*s.*; B's, £64. 12*s.* 4. £61. 10*s.* $2\frac{1}{2}$*d.* 5. A's, £125; B's, £187. 10*s.*; C's, £312. 10*s.* 6. £45; £22. 10*s.*; £30.
7. £1. 7*s.* 6*d.*; £26. 8*s.*; £3. 17*s.* 8. A's, £111. 13*s.*; B's, £130. 10*s.*
9. A's, £99; B's, £66; C's, £148. 10*s.* 10. $604\frac{1}{2}$ *lbs.*; 93 *lbs.*; $77\frac{1}{2}$ *lbs.* 11. £141. 4*s.* 3*d.* 12. 6 *mo.* 13. 9 *mo.* 14. 5 *mo.*
15. $6\frac{2}{7}$ *mo.* 16. $1\frac{5}{7}$ *dys.* 17. 5 *dys.* 18. A, £450; B, £750; C, £1200.
19. A's, £4. 14*s.* 6*d.*; B's, £5. 10*s.* 3*d.*; C's, £6. 6*s.*
20. A's, £16. 17*s.* 6*d.*; B's, £15.; C's, £13. 10*s.*

Ex. 56.—*Page* 143.

1. 8. 2. 2. 3. $28\frac{1}{4}$. 4. $16\frac{2}{3}$. 5. 15·86; 19·6; 13·33. 6. $47\frac{7}{64}$ *miles.*

Ex. 57.—*Page* 145.

1. 11; 13; 17; 25; 34; 97; 108. 2. 571; 809; 3281; 51473.
3. 6700304; 34512; 84050. 4. 673047; 84376295047.
5. ·23; ·67; ·321; ·071; ·001. 6. $\frac{1}{2}$; $\frac{5}{8}$; $\frac{13}{27}$; $\frac{61}{101}$; $\frac{3}{4}$.
7. 9; 19; 65; 312; 719. 8. 4125; 6043; 80090.
9. ·5; ·07; ·32; ·0624. 10. $\frac{1}{2}$; $\frac{3}{8}$; $\frac{9}{18}$; $1\frac{1}{2}$.
11. 5·83952. 12. 18·4824. 13. 603 *yds.* 14. 8 *ft.* 9 *in.*

Miscellaneous Exercises.

1.

1. 488914360212; 673402. 2. £21822. 11*s.* 3½*d.* 3. 379 *times.*
4. 275005 *oz.* 5. 13. 6. 6 *days.*
7. £17206 16*s.* 8*d.* 8. 33*s.*; 22*s.*

2.

1. 10000. 2. £2. 14*s.* 3*d.* 3. 2½ *miles.*
4. 5040. 5. £90. 17*s.* 2½*d.* 6. £1. 16*s.* 4*d.*
7. £30. 3 *fl.* 7 *m.* 8. 16$\frac{11}{13}$.

3.

1. £2. 13*s.* 1½*d.* 2. £1. 6*s.* 8½*d.*; £3. 6*s.* 9½*d.* 3. £10. 0*s.* 6½*d.*
4. £25. 2*c.* 5*m.* 5. 7½ *p.c.* 6. 2.
7. £273. 12*s.* 7½*d.* 8. £33. 1*s.* 6*d.*

4.

1. £4021. 14*s.* 3*d.* 2. £3. 19*s.* 8*d.* 3. 1596·71875.
4. 288 *yds.* 5. 11$\frac{11}{625}$. 6. 6¾ *p.c.*
7. £16. 6*s.* 8*d.* 8. 30 *p.c.*

5.

1. 4598077856524. 2. 614032705. 3. £51312. 8*s.* 2¾*d.*
4. £17. 19*s.* 1½*d.* 5. 359. 6. 425 *lbs.* 4 *oz.* 11 *dwt.* 5 *grs.*
7. 26. 8. 10080.

6.

1. 44731691294103. 2. 84739625049. 3. £8572 8*s.* 2½*d.*
4. 55 *tons.* 18 *cwt.* 3 *qrs.* 15 *lbs.* 5. £61. 5*s.* 4½*d.* 6. 8097.
7. £14. 3*s.* 1½*d.* 8. 140 *ac.* 1 *ro.*

7.

1. 8789475036. 2. £329. 11*s.* 5½*d.* 3. 863.
4. £1. 8*s.* 8¾*d.* 5. £4169. 6*s.* 6*d.* 6. £23. 17*s.* 2½*d.*
7. £21. 5*s.* 3*d.* 8. 0¾.

8.

1. 6470010325. 2. 171. 3. 1511640.
4. $15\frac{3}{13}$. 5. $9\frac{103}{180}$. 6. $3\frac{7}{70}$.
7. £78. 8. 5 *tons*.

9.

1. $6\frac{3}{4}$ *miles*. 2. £121. 12*s*. 6*d*. 3. 347.
4. £431. 11*s*. $6\frac{3}{4}d$. 5. $5\frac{11}{17}$. 6. £80.
7. $33\frac{1}{3}$. 8. $1\frac{1}{2}$.

10.

1. 183. 2. 153 *lbs*. 1 *oz*. 16 *dwt*. 5 *grs*. 3. $8\frac{1}{3}$, *or* $8\cdot\dot{3}$.
4. $2\frac{1}{4}$. 5. £4. 15*s*. $7\frac{1}{2}d$. 6. $123\cdot\dot{5}29411764705882\dot{3}$
7. £10. 1*s*. $0\frac{1}{2}d$. 8. £120. 6*s*. 3*d*.

11.

1. 4132. 2. £2. 16*s*. $7\frac{1}{2}d$. 3. 10 *days*.
4. 8*s*. 4*d*. 5. $2\frac{3}{4}$. 6. £122. 11*s*. $6\frac{3}{4}d$
7. £4550. 8. 5*s*. 9*d*.

12.

1. £72197. 1*s*. $8\frac{1}{4}d$. 2. 1 *hr*. 5 *m*. 3. $9\frac{3}{26}$.
4. $4\frac{1}{2}d$. 5. $\frac{1}{15}$. 6. 40 *min*.
7. £189. 3*s*. 8. $2\cdot\dot{7}70562\dot{}$.

13.

1. 2*s*. 8*d*. 2. £6596. 3. $2\frac{3}{4}$ *yds*.
4. $\frac{1}{168}$. 5. $13\frac{33}{140}$. 6. 12*s*.
7. $\frac{5}{8}$. 8. £41. 13*s*. 4*d*.

14.

1. £10066. 1*s*. $3\frac{3}{4}d$. 2. £11. 19*s*. $0\frac{3}{4}d$. 3. ·0061104.
4. $1\frac{1}{10}$. 5. 5 *p.c.* 6. 2 *p.c.*
7. £192. 7*s*. 6*d*. 8. £20. 16*s*.

15.

1. £690. 12*s*. 6*d*. 2. 48 *yrs*. 3. $3\frac{111}{121}$.
4. 59049. 5. 6*s*. $1\frac{1}{2}d$. 6. 45*s*.
7. $14\frac{1}{9}$. 8. $\cdot 21\dot{7}$.

16.

1. £5. 9*s*. 6*d*. 2. $\cdot 008325$. 3. $84\cdot 82\dot{1}42857\dot{}$.
4. 20 *p.c.* 5. 18*s*. 6. 2 *lbs*.
7. A's share = £4. 9*s*. $7\frac{1}{2}d$.; B's share = £17*s*. 18*s*. 5*d*.; C's share = £8. 19*s*. $2\frac{1}{2}d$. 8. £725. 13*s*. 6*d*.

17.

1. $4\frac{1}{3}$ *days*. 2. $\frac{8}{21}$. 3. X = 12*s*.; Y = 10*s*.; Z = 15*s*.
4. £342. 2*s*. 6*d*. 5. 1 *hr*. 42 *min*. 6. 101.
7. £4. 5*s*. 8. $9\frac{8}{13}$.

18.

1. £21. 3*s*. $6\frac{3}{4}d$. 2. $\frac{1}{3}$. 3. £2. 12*s*.
4. £1700. 5. 20 *p.c.* 6. A's = £112; B's = £126; C's = £120. 7. 2. 8. $3\frac{1}{7}$.

19.

1. £140. 12*s*. 6*d*. 2. 4*s*. $3\frac{3}{4}d$. 3. $6\frac{1}{13}$.
4. 15*s*. 5. 17*s*. $1\frac{1}{2}d$. 6. 204.
7. £5. 13*s*. 4*d*. 8. £3. 11*s*. 8*d*.

20.

1. 8*s*. $5\frac{1}{2}d$. 2. £1238. 17*s*. $4\frac{1}{2}d$. 3. 2 *hrs*.
4. 2·1. 5. $1\cdot 0\dot{0}\dot{9}$. 6. $7\frac{1}{2}$ *hrs*.
7. $2\frac{1}{2}$ *yrs*. 8. £100.

21.

1. C. 2. $\cdot 18\dot{3}$. 3. £4680.
4. 12 *ft*. 5. £100. 6. £1464. 2*s*.
7. £1. 12*s*. 3*d*. 8. 369007.

22.

1. 8*s.* 5½*d.* 2. A, 35*s.*; B, 25*s.* 3. £71*s.* 14*s.* 2¼*d.*
4. ⅛. 5. 1*s.* 6. £857. 10*s.*; £1429.
£2000. 16*s.* 8*d.* 7. 4*s.* 8. £166. 13*s.* 4*d.*

23.

1. 8. 2. ·578703. 3. 2 *tons.* 3 *qrs.*
4. £10*s.* 6*s.* 8*d.* 5. ¼. 6. 20 *per cent.*
7. 121. 8. £26. 13*s.* 4*d.*

24.

1. £1003. 8*s.* 2*d.* 2. 15 *p.c.* 3. £171. 19
4. 3¾ *days.* 5. 11 *ac.* 4 *po.* 6. ·242133;
7. 5 *men.* 8. £21. 4*s.* 3$\frac{1}{13}$*d.*

25.

1. £56429. 10*s.* 9¼*d.*; 7 *cwt.* 3 *qrs.* 27 *lbs.* 2. 37896. 3. 197
4. 6. 5. £155. 2*s.* 9*d.* 6. ·00
7. 1¼*d.* 8. 9⅜ *sq. yds.*

26.

1. 15. 2. 1$\frac{173}{305}$. 3. $\frac{5}{21}$.
4. 15625.; 625. 5. 6¼ *p.c.* 6. £¼.
7. £560. 8. 5*s.* 0$\frac{8}{61}$*d.*

27.

1. £2. 19*s.* 7*d.* 2. 16·473. 3. 2·380
4. Loss £3. 10*s.* 5. 1⅛. 6. 300.
7. 28 *feet.* 8. ⅛ or ·125.

28.

1. £4. 1*s.* 2. £12. 19*s.* 3. 789¾ *s*
4. 1¼. 5. $\frac{5}{16}$ *p.c.* 6. £5. 5*s*
7. £1. 14*s.* 2¼*d.* 8. £860. 5*s.*



